

Correlation Analysis of Wind and Photovoltaic Power Based on Mixed Copula Theory and Its Application into Optimum Capacity Allocation

Shitian Xu

North China Electric Power University
State Key Laboratory of Alternate Electrical Power System with
Renewable Energy Sources
Beijing, China
shitian_xu2017@163.com

Chenbo Su

North China Electric Power University
State Key Laboratory of Alternate Electrical Power System with
Renewable Energy Sources
Beijing, China
373607885@qq.com

Chongru Liu*

North China Electric Power University
State Key Laboratory of Alternate Electrical Power System with
Renewable Energy Sources
Beijing, China
chongru.liu@ncepu.edu.cn

Chao Wang

North China Electric Power University
State Key Laboratory of Alternate Electrical Power System with
Renewable Energy Sources
Beijing, China
huadianwch@163.com

Abstract—In order to better realize the rational utilization of renewable energy and reduce the large-scale and multi-type renewable energy's output for the stable operation of power grid, an optimum capacity allocation method which is based on the correlation between wind and photovoltaic (PV) power output, is proposed. In this paper, the historical data of actual wind speed and PV power output in northern Hebei are analyzed. Based on this, a mathematical model of correlation analysis using mixed copula function is established by combining original least square method. Moreover, the Spearman rank correlation coefficient is selected to quantitatively measure the generation characteristics of the two kinds of fluctuating energy output, and the correlation calculation based on wind and PV power output is performed to obtain the optimum capacity allocation scheme.

Keywords—wind power, PV power, mix copula theory, correlation, optimum capacity allocation

I. INTRODUCTION

In recent years, renewable energy, represented by wind and photovoltaic (PV) power, has become an important part of the power grid. The renewable energy is characterized by intermittent, fluctuating, and random nature [1]; On the other hand, planning of the renewable energy generation system does not match well with construction of the grid [2-3]. Therefore, the phenomenon of wind and PV power curtailment often occurs when such type of renewable energy sources are integrated into the power grid. In order to improve the utilization of transmission lines effectively and enhance the economy of multi-type renewable energy generation system, research on the optimum capacity allocation of wind and PV power have been widely concerned.

According to the complementary characteristic of wind and PV power output in space and time, scholars study the optimum capacity allocation [4-8]. Literature [4] calculated the correlation between wind and PV power with the original output data directly. Literature [5] aimed at to minimize the variation of peak-valley load difference through simulation.

In [6], with the goal of minimizing the relative standard deviation of wind-PV composite output, the corresponding optimal capacity allocation is obtained, but the wind-PV composite output was just a simple superposition. In [7-8], from the perspective of multi-objective optimization, considering the constraints of capacity allocation, the complementary characteristics are used to reflect the reliability of the power grid.

However, the current research in this field, lacks in wind-PV combined power output characteristics in detail for capacity allocation method. Accordingly, the copula function is applied to the study of the correlation between wind and PV power output [9-11]. It is verified that the correlation probability sequence generated by copula function can be used for the modeling of wind farm and PV power station effectively. However, the output characteristics of PV power are different from that of wind power, which has obvious tail characteristics. A single copula function cannot completely describe the correlation of wind-PV joint power output.

Therefore, this paper proposes a new method based on mix copula theory for calculating the correlation of wind-PV joint power output, and obtains the optimum capacity allocation scheme. The organization of this article is as follows. In Section II, basic copula theory is introduced and the mixed copula model is employed. In Section III, output characteristics of the renewable energy are explored, and in order to fit wind-PV joint distribution, a parameter estimation model of mixed copula function is established based on ordinary least-square method. Moreover, the correlation of wind-PV joint power output is calculated, and a scheme of optimum capacity allocation is proposed for the purpose of optimizing its complementary characteristics. A case study is presented in Section IV, practical historical data in northern Hebei are applied to verify the proposed schemes. Section V concludes the findings.

II. MIX COPULA THEORY

A. Basic copula theory

Copula function describes the correlation between multiple variables, which can be used to measure the correlation between continuous random variables. Copula function has the greatest advantage of solving complex high-dimensional joint probability distribution. Applying copula theory, multivariable joint distribution of wind and PV power output can be separated into univariate marginal distribution and a connection function that connect these correlation marginal distribution^[12-13].

Copula function is a connection function. Every cumulative distribution function $F(x_1), F(x_2), \dots, F(x_N)$ can be expressed in terms of its marginal distribution function $F_1(x), F_2(x), \dots, F_N(x)$. When the marginal distribution is continuous, the copula function can be uniquely determined and satisfies:

$$F(x_1, x_2, \dots, x_n) = C(F_1(x_1), F_2(x_2), \dots, F_N(x_N)) \quad (1)$$

Rank correlation coefficient and tail correlation coefficient of copula can describe the nonlinearity, asymmetry, upper and lower tail relationship between wind-PV joint power better^[14-15].

B. Maintaining the Integrity of the Specifications

Copula functions include three types: multivariate normal copula, multivariate T-copula and Archimedes copula. Additionally, Archimedes copula mainly contains Gumbel copula, Clayton copula and Frank copula^[16]. The characteristics of different Copula functions are listed in Table I.

TABLE I. THE CHARACTERISTICS OF DIFFERENT COPULA

Copula Function	Characteristic
Normal copula	Symmetric distribution and no thick tail
T-copula	Symmetric distribution and certain thick tail
Gumbel copula	Asymmetric distribution and upper-tail
Clayton copula	Asymmetric distribution and lower-tail
Frank copula	Negative and symmetric distribution

Mixed copula function is composed of different copula functions multiplied by weight coefficients. Compared with a particular copula function, the mixed copula function can reflect the correlation and asymmetric structure of non-linear variables more accurately.

III. MODELING OF CORRELATION AND OPTIMUM CAPACITY ALLOCATION OF WIND-PV JOINT POWER OUTPUT

A. Output characteristics of the renewable energy

Based on actual historical data of wind speed and PV power output of a wind farm and PV power plant in northern Hebei Province, which is sampled with interval of 10 minutes. The characteristics of wind and PV power output on typical days in summer and winter can be obtained, as shown in Fig.1.

It can be observed that the wind power output have

This work is supported by the Science and Technology Program of State Grid Corporation of China/Headquarters Research on key technology of series sub-synchronous oscillation suppressor. Project No: GEIRI-DL-71-18-005.

obvious characteristics of peak reversal. In daytime for s=summer (peak load), the wind speed is small, and in night time for winter (load valley) the wind speed is high. PV power output is related to light intensity and sunshine conditions, and has obvious seasonal difference. Compared with winter, the output is increased in summer, the period of PV power output becomes longer. The peak period of PV output in summer is generally 11 h to 13 h, while in winter it is 12 h to 14 h, and there is almost no PV power output at night.

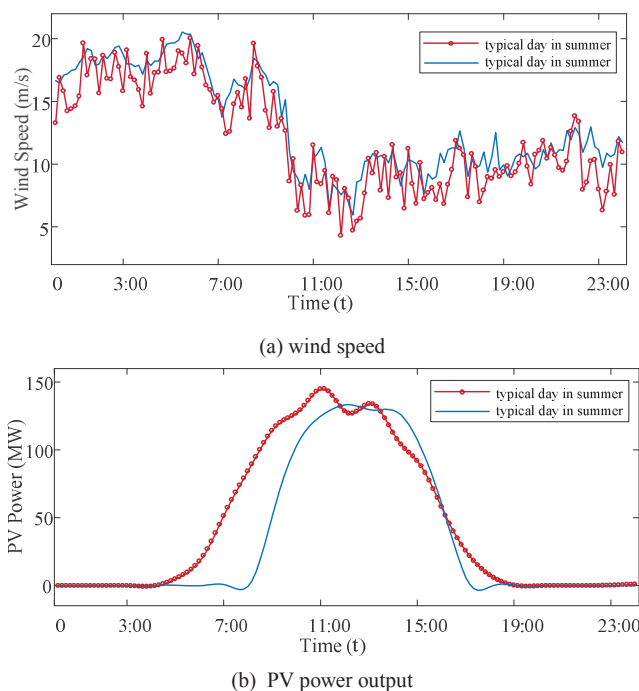


Fig. 1. Comparison of typical PV power and wind speed distribution in summer and winter in northern Jibe

B. Modeling and parameter estimation of the mixed copula

According to the characteristics of wind and PV power output mentioned above, this paper selects Gumbel copula, Clayton copula and T-copula to establish a mixed copula function for modeling wind-PV joint distribution, as shown in (2).

$$C_M(P_w, P_v) = \omega_1 C_G(P_w, P_v; \theta_1) + \omega_2 C_c(P_w, P_v; \theta_2) + \omega_3 C_T(P_w, P_v; \theta_3) \quad (2)$$

Where C_G , C_c and C_T represent Gumbel, Clayton copula and T-copula function respectively; P_w and P_v represent wind and PV power output respectively; θ_1 , θ_2 and θ_3 are the parameter of different copula function respectively; ω_1 , ω_2 and ω_3 are weight coefficient of different copula function respectively, satisfying $\omega_1 + \omega_2 + \omega_3 = 1$, and reflecting the wind-PV joint distribution characteristics.

In this paper, ordinary least squares (OLS) method is used to estimate the parameters in (2), and its necessary steps are shown as follows:

Step1: For function expression (3), its matrix form can be expressed as (4).

$$h_\omega(x_1, x_2, \dots, x_n) = \omega_0 + \omega_1 x_1 + \dots + \omega_n x_n \quad (3)$$

$$x^{(i)} = \begin{pmatrix} x_0^{(i)} \\ x_1^{(i)} \\ \vdots \\ x_n^{(i)} \end{pmatrix} \quad \omega = \begin{pmatrix} \omega_0 \\ \omega_1 \\ \vdots \\ \omega_n \end{pmatrix} \quad X = \begin{pmatrix} (x^{(1)})^T \\ (x^{(2)})^T \\ \vdots \\ (x^{(m)})^T \end{pmatrix} \quad (4)$$

Step2: Define the expression of loss function as (5). In that way, the calculation formula of applied matrix can be described as (6).

$$J(\omega) = \frac{1}{2} \sum_{i=1}^m (h_{\omega}(x^{(i)}) - y^{(i)})^2 \quad (5)$$

$$= \frac{1}{2} \text{tr}[(X\omega - Y)^T (X\omega - Y)]$$

$$\frac{\partial J(\omega)}{\partial \omega} = \frac{1}{2} \cdot \frac{\partial \text{tr}(\omega^T X^T X\omega - \omega^T X^T Y - Y^T X\omega + Y^T Y)}{\partial \omega} \quad (6)$$

$$= X^T X\omega - X^T Y$$

Step3: The expression of the weight coefficient matrix can be obtained when the value of (6) is equal to 0, as shown in (7). Since that θ_i is a parameter in a variable X and it is independent of the distribution ω , the maximum likelihood estimation can be derived.

$$\omega = (X^T X)^{-1} X^T Y \quad (7)$$

To ensure the accuracy of calculation, empirical copula function is introduced to estimate the joint distribution function of actual wind and PV power output. Assuming that (x_i, y_i) is taken from wind and PV power output sequences (P_w, P_v) , their marginal distribution functions are $F_n(P_w)$ and $G_n(P_v)$ respectively, and the sample experience copula is defined as follows:

$$\hat{C}_n(P_w, P_v) = \frac{1}{n} \sum_{i=1}^n I_{[F_n(x_i) \leq P_w]} I_{[G_n(y_i) \leq P_v]} \quad (8)$$

$$I = \begin{cases} 1 & F_n(x_i) \leq P_w, G_n(y_i) \leq P_v \\ 0 & F_n(x_i) > P_w, G_n(y_i) > P_v \end{cases}$$

Where $I[\cdot]$ is a indicator function. Substituting (2) and (8) into (5), the estimate expression of empirical copula function can be derived, as shown in (9).

$$\ell(\omega; \theta) = -\frac{\beta}{2} \sum_{i=1}^n [\hat{C}_n(P_w, P_v) - C_M(P_w, P_v)] \quad (9)$$

Where θ and ω are the parameters of the copula function, and independent of each other. Accordingly, computing the partial derivative of (9) with respect to θ and ω , as shown in (10). Then the value of these parameters can be calculated.

$$\frac{\partial \ell(\omega; \theta)}{\omega} = 0, \quad \frac{\partial \ell(\omega; \theta)}{\theta} = 0 \quad (10)$$

C. Optimum capacity allocation based on correlation calculation of wind-PV joint power output

Based on the above analysis, the correlation of wind-PV joint power output can be calculated. With the goal of optimizing complementary characteristics, optimum capacity allocation scheme can be obtained. The flow chart is shown in Fig. 2.

Step1: Reading historical wind speed and PV power data. Convert the actual output of 8760 hours a year into a standard unit value. Using (11), the wind power output can be calculated from the wind speed.

$$P_w = \begin{cases} 0 & v < v_{wi}, v > v_{wo} \\ v^3 - v_{wi}^3 & v_{wi} < v < v_r \\ v_r^3 - v_{wi}^3 & P_r & v_r \leq v < v_{wo} \\ P_r & \end{cases} \quad (11)$$

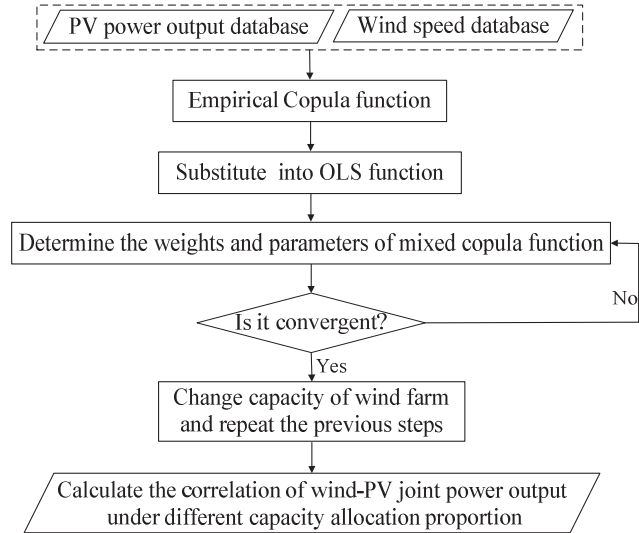


Fig. 2. Calculation flow chart

Where v_{wi} , v_{wo} , and v_r are the cut-in, the cut-out, and the rated wind speed, respectively; P_r is the rated active power output for a single wind farm. Referring to the actual wind farm, set v_{wi} as 3 m/s, v_{wo} as 24 m/s and v_r as 13 m/s.

Step2: Determining the marginal distribution of wind and PV power through nonparametric kernel density estimation method.

Assume that X_1, X_2, \dots, X_N are taken from discrete calculated wind power output. Nuclear density estimation of arbitrary wind power output x can be defined as (12). As for PV power output, it can be obtained similarly.

$$\hat{f}_h(x) = \frac{1}{nh} \sum_{i=1}^n K\left(\frac{x - X_i}{h}\right) \quad (12)$$

Step3: Establishing mixed copula model to fit the probability density function(PDF) of wind-PV joint power output. The goal of this step is to determine the weights and parameters for each copula through OLS method which has mentioned above.

Step4: Selecting Spearman rank coefficient to describe the complementary characteristics of the new wind and PV power output series. Using (13), calculate the Spearman coefficient of the different capacity allocation proportion under different seasonal conditions. Consequently, optimum capacity allocation scheme can be obtained.

$$\hat{\rho}_s = 1 - \frac{6}{n(n^2 - 1)} \sum_{i=1}^n (R_i - Q_i)^2 \quad (13)$$

Where $\hat{\rho}_s$ is Spearman coefficient; R_i and Q_i are the rank of different sample, respectively.

IV. CASE STUDY

In order to verify the effectiveness of the method shown in Fig.1, wind speed and PV power output data from geographical locations of Zhangnan wind farm and Yunjushan PV power station in northern Hebei are tested. Concretely, results of the proposed method are as follows.

1) The joint frequency histogram of distributed wind and PV output marginal value are presented in Fig.3. Conclusion can be drawn that with the increase of wind farm capacity, the marginal distribution gradually convex and the middle distribution gradually concave.

2) Fig.4 shows the PDF histogram of wind-PV joint power distribution obtained by using the empirical copula distribution. Compared with the previous results shown in Fig.3, the above results have highly similar feature information, and thus the proposed method is validated in describing the correlation characteristics of wind-PV joint power.

To further explore the advantages of the proposed method, T-copula function and mixed copula function with OLS method are used to calculate the correlation of wind-PV joint power output. Their standard deviation and the distribution difference are obtained through the K-S test method. All results of the K-S test are listed in Table II. The significant level selected is 5% where K represents the detection index. The smaller the value of k, the smaller the difference between the cumulative distribution functions.

In addition, T-copula function and mixed copula function with OLS method are compared with empirical copula function respectively. The quantile quantile(q-q) plot in Fig.5 used the dataset of two sets of functions mentioned above. It intuitively shows the similarity of probability density functions distribution coming from different data sets.

Fig. 3. Joint distribution histogram of different capacity proportion

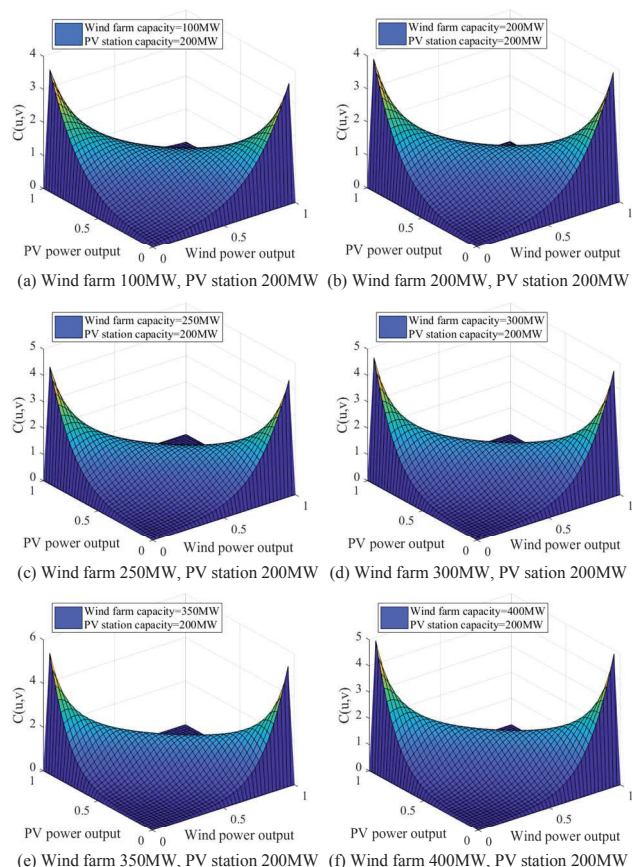


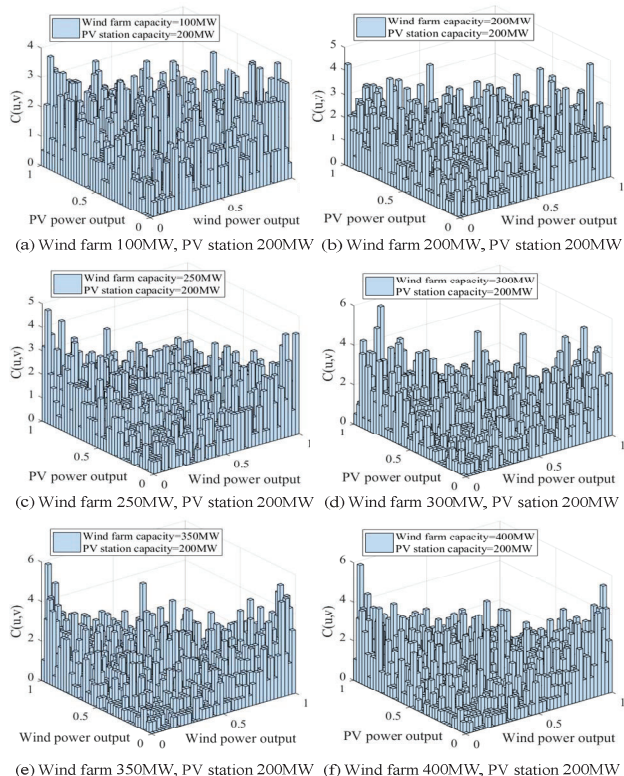
Fig. 4. Function probability density diagram of mixed Copula for different capacity proportion

When the value of Y quantiles or the departure from the reference line is smaller, the difference between the cumulative distribution functions is smaller.

From Table II and Fig.5, it can be observed that the K-S detection value, Y quantiles and the departure from the reference line is the smallest, when the mixed copula function with OLS method is adopted to fit the wind-PV joint distribution. These validation results reveals that the correlation of wind-PV joint power output calculated by the proposed method is more accurate and more satisfactorily agree with the actual situation. K-S Detection Results of the two Algorithms for Different Capacity Proportion

TABLE II. K-S DETECTION RESULTS OF THE TWO ALGORITHMS FOR DIFFERENT CAPACITY PROPORTION

PV station capacity/MW	Wind farm capacity/MW	K $C_{LMS}(v_1, v_2)$	k $C_T(v_1, v_2)$
200	100	0.287	0.354
200	200	0.283	0.356
200	250	0.279	0.348
200	300	0.275	0.345
200	350	0.273	0.344
200	400	0.278	0.349



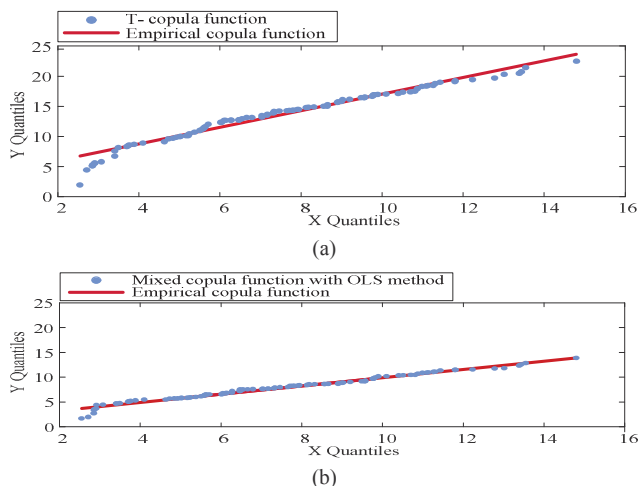


Fig. 5. Quantile quantile plot

TABLE III. CORRELATION CALCULATION OF WIND-PV JOINT POWER OUTPUT FOR DIFFERENT CAPACITY PROPORTION

PV station capacity/MW	Wind farm capacity/MW	Spearman rank coefficient	
		Summer	Winter
200	100	-0.2227	-0.2433
200	200	-0.2764	-0.2882
200	250	-0.3041	-0.3196
200	300	-0.3337	-0.3467
200	350	-0.3628	-0.3756
200	400	-0.3402	-0.3518

3) Table III shows the correlation calculation of wind-PV combined power output for different capacity proportion by maintaining PV capacity of 200 MW and setting different wind farm capacity.

As illustrated in Table III, Spearman rank coefficient is negative in general, which indicates that the complementary characteristic of wind and PV power output, and the complementarity in winter is higher than that in summer specifically. It can be concluded that Spearman coefficient increases continuously when the wind capacity is between 100 MW and 350 MW, but decreases when the wind capacity is 400 MW, which indicates that the optimum proportion of wind/PV power capacity is between 1.75-2.0. Through multiple simulations, it is verified that Spearman coefficient reaches at maximum when the wind farm capacity is 368 MW, which is equal to -0.3749 in summer and -0.3851 in winter. Therefore, the optimum proportion of wind/PV power capacity in northern Hebei is 1.84.

V. CONCLUSIONS

This paper analyses the correlation of joint wind-PV power output based on mixed copula function model combined with OLS method. The optimum capacity allocation scheme is explored with the intention of getting the best complementary characteristics. The following conclusions can be made after performing above analysis:

1) Mixed copula function model with OLS method behaves better than single copula, and it can improve the description of the correlation structure for the joint wind-PV power output in actual situation.

2) The optimum proportion of wind and PV power capacity is 1.84 in northern Hebei. However, capacity of integrated wind power is 94.334 million kilowatts, and capacity of integrated PV power is 18.225 million kilowatts in northern Hebei at present. The proportion of wind/PV power capacity is far greater than the optimum value. Therefore, it is suggested that PV power plants in northern Hebei should be appropriately increased in future power grid construction.

3) Research of the optimum capacity allocation by using complementary characteristics can make better use of the intrinsic wind and PV resource, and provide a theoretical basis and confidential reference for power system operation and plan.

REFERENCES

- [1] M. Marinelli, P. Maule, A. N. Hahmann, O. Gehrke, P. B. Nørgård and N. A. Cutululis, "Wind and Photovoltaic Large-Scale Regional Models for Hourly Production Evaluation," in IEEE Transactions on Sustainable Energy, vol. 6, no. 3, pp. 916-923, July 2015.
- [2] X. D. Liu, K. Fang, H. Y. Chen and C. Q. She, "Research on rational wind power casting theory for large-scale wind power integration improvement," Power System Protection and Control, vol. 40, no. 6, pp. 36-39, 2012.
- [3] H. M. Li, J. X. Luo and Q. L. Wan, "Manual complementarity optimization of electrical source involving intermittent power," Power System Protection and Control, 2013, 41(9): 34-39.
- [4] L. Pan, L. X. Xu, Z. Zhuan and X. L. Qin, "Study on the Proportion of New Energy Capacity Based on Wind Power and Photovoltaic Power Correlation Analysis," Telecom Power Technology, 2016, 33(2): 1-3.
- [5] C. Y. Sun, L. Wei, H. G. Jia and L. Xin, "Discussions on the Theoretical Installed Proportion of Wind Power and Photovoltaic Power in New Energy Development," Power System and Clean Energy, vol. 30, no. 6, pp. 24-32, 2014.
- [6] S. J. Wang, X. H. Wang, P. Ren and J. Jiao, "The applied research of the wind and solar hybrid project," Solar Energy, 2013(8): 56-60.
- [7] L. Xu, X. B. Ruan, B. H. Zhang and C. X. Mao, "An improved optimal sizing method for wind-solar-battery hybrid power system," Proceedings of the CSEE, 2012, 32(25): 88-98.
- [8] T. L. Yao, X. Q. Wu, Z. W. Li, X. Li and K. Wang, "Research on complementary capacity ratio of wind power and photovoltaic considering multiple constraints," Power System Protection and Control, 2017.
- [9] Papaefthymiou G and Kurowicka D, "Using Copulas for Modeling Stochastic Dependence in Power System Uncertainty Analysis," IEEE Transactions on Power Systems, 2009, 24(1): 40-49.
- [10] Bruce Stephen, "A Copula Model of Wind Turbine Performance," IEEE Transactions on Power Systems, 2011, 26(2): 965-966.
- [11] M. Wang, X. T. Mao, H. M. Wang and Y. Yuan, "Reliability analysis of generation systems with photovoltaic stations," Proceedings of the CSEE, 2013, 33(34): 42-49.
- [12] Y. Q. Xu, K. Chen, J. Q. Li and Y. Nie, "A New Method Analyzing Output Correlation of Multi-Wind Farms Based on Combination of Copula Function and Kernel Estimation Theory," Transactions of China Electrotechnical Society. 2016, 31(13): 92-100.
- [13] Y. Wang and Y. Luo, "Research of Wind Power Correlation With Three Different Data Types Based on Mixed Copula," in IEEE Access, vol. 6, pp. 77986-77995, 2018.
- [14] Ji F, Cai X G, Wang J, "Wind Power Correlation Analysis Based on Hybrid Copula," Automation of Electric Power Systems, 2014, 38(2): 1-5.
- [15] S Morozumi, K. Nara. "Recent trend of new type power delivery system and its demonstrative project in Japan", IEEE Transactions on Power and Energy, vol. 127, no. 7, pp. 770-775, 2007.
- [16] X. Pan, L. L. Wang, Y. Q. Xu, L. Zhang and W. X. Liu, "A Wind Farm Power Modeling Method Based on Mixed Copula," Automation of Electric Power Systems. 2014, 38(14): 17-22.