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Least Square Method: A Novel Approach to Determine Symmetrical Components of Power System

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Abstract – This paper proposes a novel approach to determine symmetrical components of power system by applying method of least squares in time domain. For the modern power system stability, clearance of faults on high voltage transmission lines in zero response time is crucial and important. Symmetrical components have a great attention since last century. They have been found an effective tool for the analysis of symmetrical and unsymmetrical faults in power system. Moreover, magnitude of symmetrical components are also used as a caution about faults in system. With rapid changes in technology, Microprocessor assumed to be fastest machine of the modern era. Hence microprocessor based techniques were developed and implemented for last few decades. The proposed technique apply least square method in the computation of symmetrical components which is suitable as an application in microprocessor based monitoring and controlling power system in order to avoid cascading failures. Simulation of proposed model is carried out in MATLAB/SIMULINK and all results exploit the validity of model.

Keyword: Digital protection system, Least square method, Symmetrical components, MATLAB, SIMULINK

1. Introduction

Various technologies have been developed in past to implement protective devices that correctly detect disturbance in power system and take remedial steps. Conventional protection relays use electromagnetic principle for their operation [1]. These relays energized when magnitude of operating signal becomes larger than the magnitude of the threshold signal. These relays were categorized as amplitude comparators. The response time of these electromagnetic relays was large. To overcome this problem, solid state relays were introduced. Solid state relays were fast and static i.e no moving parts have been designated to carry out their duties required.

In past few decades, rapid growth of computer technologies led the researcher to design a computer based technique to implement protection system [2]. Microprocessor based relays have many advantages over conventional relays [1]. However the basic protection principles have remained principally unchanged throughout in advanced microprocessor based relays [3].

Transmission lines in the power system components have most fault incident rate because they lie in open environment and weather conditions affect it badly. Line faults are mostly caused by lightening, fog, thunder, tree fall and many more, which are beyond human control. Broadly speaking, the faults are categorized as balance and unbalance faults.

3ϕ shunt and 3ϕ to ground faults are classified as balance faults while single line to ground, line to line and double line to ground faults are unbalance faults in power system.

In power system protection, digital techniques have been considered effective as compared to conventional analogue techniques. Digital protection techniques receive voltage and current signals from acquisition system [4].

Algorithm implemented in digital protection system has great importance. Many researchers have developed algorithms to efficiently detect and solve power system problems [5]. Symmetrical components exploits the faults in system more precisely and rapidly [6]. In this paper, a microprocessor based technique using least square method to determine symmetrical components have been developed and analyzed.

2. Symmetrical Component Calculation

Symmetrical component decomposition is a valuable technique presented by Fortescue to solve balance and unbalance power system in terms of equivalent symmetrical components [7]. The unbalance power system is complex to understand so it is difficult to investigate

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problem occurred due to abnormal conditions. The transformation of unbalance power system into balance components makes the investigation simple and fast. The unbalance 3ф system can be written as [8]

$$\begin{bmatrix} V_a(t) \\ V_b(t) \\ V_c(t) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix} \begin{bmatrix} V_a{}^0(t) \\ V_a{}^1(t) \\ V_a{}^2(t) \end{bmatrix} \quad (1)$$

$V_a{}^0$, $V_a{}^1$ and $V_a{}^2$ are zero, positive and negative sequence components respectively [8] [9]. The Eq. (1) can be mathematically solved as

$$\begin{bmatrix} V_a{}^0(t) \\ V_a{}^1(t) \\ V_a{}^2(t) \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix} \begin{bmatrix} V_a(t) \\ V_b(t) \\ V_c(t) \end{bmatrix} \quad (2)$$

α is complex operator of value $e^{\frac{2\pi}{3}}$.

The instantaneous phase voltage can be expressed as

$$V_a(t) = \mathcal{R}e[V_a e^{j\omega t}]\mathrm{Z} \quad (3)$$

Assuming positive sequence network of balance 3ф system

$$V_b(t) = \alpha^2 V_a(t) = \alpha^2 \mathcal{R}e[V_a e^{j(\omega t - \frac{2\pi}{3})}] \quad (4)$$

Similarly

$$V_c(t) = \alpha V_a(t) = \alpha \mathcal{R}e[V_a e^{j(\omega t + \frac{2\pi}{3})}] \quad (5)$$

Resolving α into non-anticipating values, Eq. (6) formulate the complete process to compute symmetrical components [9] [10]. Here T is time period of phase voltage $V_a(t)$.

$$\begin{bmatrix} V_a{}^0(t) \\ V_a{}^1(t) \\ V_a{}^2(t) \end{bmatrix} = \frac{1}{3} \mathcal{R}e \begin{bmatrix} V_a(t) + V_b(t) + V_c(t) \\ V_a(t) + V_b(t - \frac{2T}{3}) + V_c(t - \frac{T}{3}) \\ V_a(t) + V_b\left(t - \frac{T}{3}\right) + V_c(t - \frac{2T}{3}) \end{bmatrix} \quad (6)$$

The protection system should be intelligent enough to isolate the faulty component from the system. For this reason, magnitude of symmetrical components must be known.

3. Magnitude of Symmetrical Components

Suppose $f(t)$ is known symmetrical components which can be expressed as [9]

$$f(t) = K \sin(\omega t + \theta) \quad (7)$$

Using trigonometric identity, Eq. (7) can be reformed as

$$f(t) = K \sin(\omega t + \theta) = K \cos\theta \sin\omega t + K \sin\theta \cos\omega t \quad (8)$$

Eq. (8) have two unknowns K cos θ and K sin θ which require minimum two equations for their solution. Expanding Eq. (8) for numerous samples results Eq. (9) as [9]

$$\begin{bmatrix} \sin\omega t & \cos\omega t \\ \sin\omega\,(t - t_s) & \cos\omega(t - t_s) \\ \sin\omega(t - 2t_s) & \cos\omega(t - 2t_s) \\ . & . \\ \sin\omega(t - (N-1)t_s) & \cos\omega(t - (N-1)t_s) \end{bmatrix} \begin{bmatrix} K\cos\theta \\ K\sin\theta \end{bmatrix} = \begin{bmatrix} f(t) \\ f(t - t_s) \\ f(t - 2t_s) \\ . \\ f(t - (N-1)t_s) \end{bmatrix} \quad (9)$$

For simplicity Eq. (9) can be written as

$$[A][x] = [f] \quad (10)$$

N is the number of samples taken for calculation and t_s is the difference of time between two samples. Eq. (10) can be solved as

$$[x] = [A]^{-1}[f] \quad (11)$$

Using basic mathematical rules, $[A]^{-1}$ can easily be computed. However for $N > 2$ pseudo-inverse method will be deployed to calculate $[A]^{-1}$. The elements of matrix A can be computed by selecting any reference time t such that elements of A^{-1} should have values close to one another as possible.

It has been observed [9] that for $N = 6$

$$A = \begin{bmatrix} -0.5010 & -0.8651 \\ -0.9990 & 0 \\ -0.5010 & 0.8651 \\ 0.5010 & 0.8651 \\ 0.9990 & 0 \\ 0.5010 & -0.8651 \end{bmatrix} \quad (12)$$

The block diagram of model described in Eq. (6)-(11)

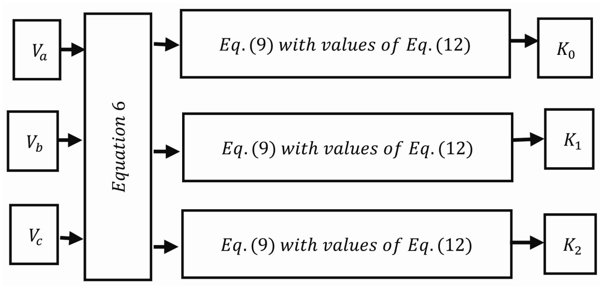

Fig. 1. Block Diagram to calculate magnitude of symmetrical components

Table 1. Magnitude of symmetrical components

Condition	Simulated(V_{rms})			Reference Figure
	Zero	Positive	Negative	
Normal	0.00	220.2	0.43	3
SLG	75.35	146.3	71.34	4
DLG	74.27	73.44	72.62	5
L-L	0.03	110.3	110.3	6
L-L-L	0.011	0.009	0.008	7

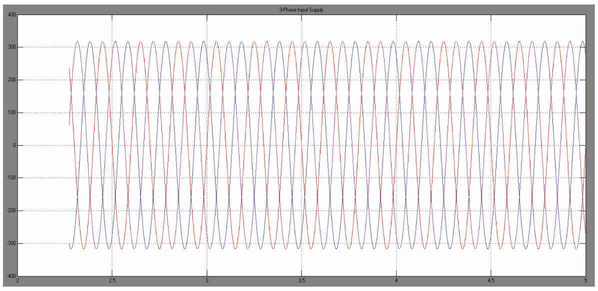

Fig. 2. 3ϕ supply to model

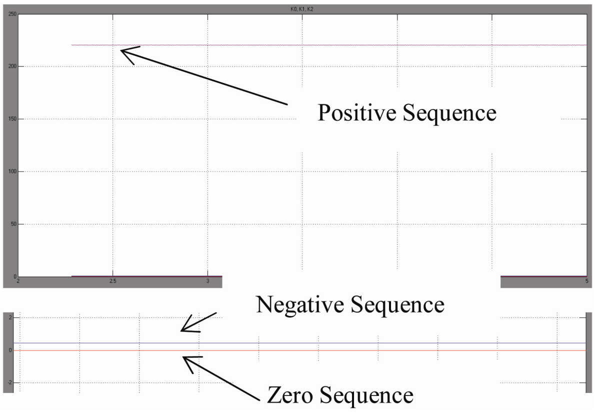

Fig. 3. Behavior of model under normal conditions

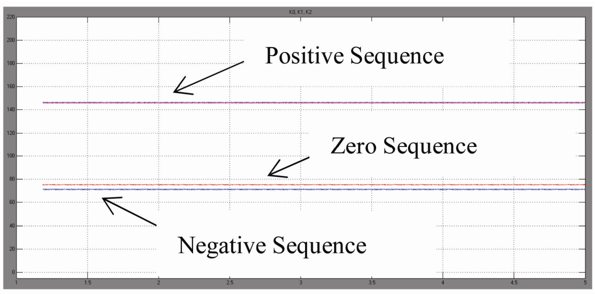

Fig. 4. Behavior of model under Single line to ground fault

can easily be extracted as Fig. 1. The input to this model is phase voltage of 3ϕ system while the output is magnitude of symmetrical components. The 3ϕ supply to the model is shown in figure 2. Figs. 3-6 and 7 present the behavior of model under normal conditions, Single line to ground fault, double line to ground fault, line to line fault and L-L-L fault respectively. The magnitude of symmetrical components are given in Table 1.

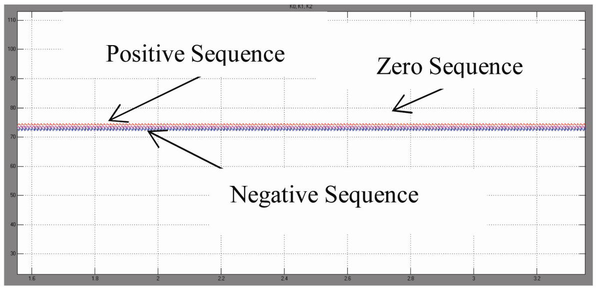

Fig. 5. Behavior of model under Double line to ground Fault

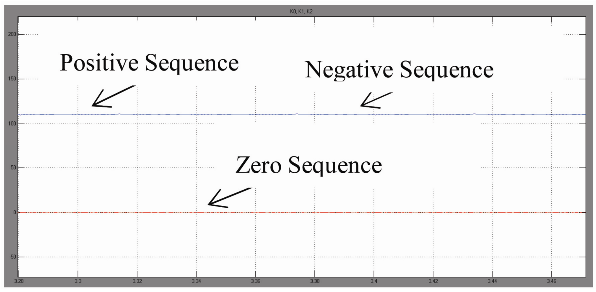

Fig. 6. Behavior of model under L-L Fault

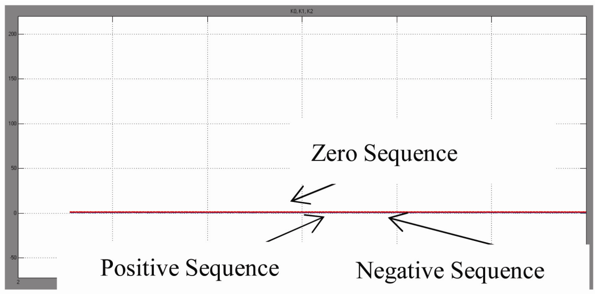

Fig. 7. Behavior of model under L-L-L Fault

4. Method of Least Squares

Method of least squares is a mathematical technique commonly used in the field of engineering and applied sciences [11-13, 14, 16]. Least square method is practiced to find line of best fit. The derivation can easily be made using basic rules of algebra. The method has significant importance in the areas of prediction, numerical analysis, communication and control [15]. The topic presents another application of least square method to estimate values of symmetrical components in power system in order to minimize the shutdown of electricity. It has been studied that the output $y(nT)$ of discrete system is given by weighted sum of present and a finite number of past values of the input $v[(n-m)T], m = 0,1,2 \ldots M-1$ as

$$y(nT) = v(nT)h(0) + v[(n-1)T]h(T) + \cdots + v[(n-M-1)T]h[(M-1)T] = \sum_{m=0}^{M-1} v[(n-m)T]h(mT) \qquad (13)$$

The optimal solution of weighting factors $h(0)$, $h(1) \dots h(mT)$ needs $J \geq M$ sets of measurements. For $J = M$, unique solution of h vector exists. But the system noise and probability of errors make the result incorrect. In such scenario, statistical analysis have been proved a prime method. Thus $J > M$ measurements have been taken and best value of vector h is estimated using least square method [15]. The weighting vector estimated by least square method is given as

$$h = (v^T v)^{-1} v^T y \tag{14}$$

The Eq. (14) result's the estimated values of weighting factor based on J measurements. What happens to h when new sample of v arrives after J measurements?

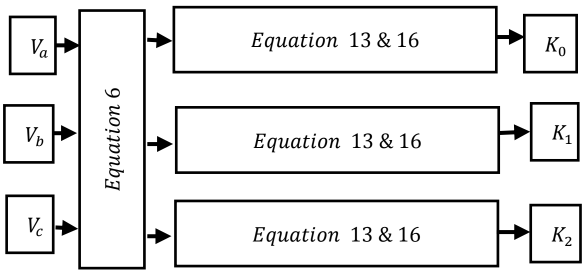

Fig. 8. Block Diagram to calculate magnitude of symmetrical components using proposed technique

Table 2. Magnitude of symmetrical components with least square algorithm

Conditions	Simulated Results of Figure 8(V_{rms})		
	Negative sequence	Positive sequence	Zero sequence
Normal	0.0163	7.2784	0
SLG	1.0108	3.6642	0.8311
DLG	19.1022	19.1022	19.1022
L-L	28.6030	28.6030	0
L-L-L	0.0169	0.0169	0.0051
1-Conductor Broken	5.7888	24.2371	6.4085
2-Conductor Broken	19.1022	19.1022	19.1022

The new values of h will be determined on $J+1$ measurements. For each case, computation involves matrix inversion which is not satisfactory [15]. Moreover to reduce time of computation it is better to add impact of newly arrived sample instead of computing h again.

Let

$$P_J = (v^T v)^{-1} \tag{15}$$

$$h_{J+1} = h_J + P_J v_{J+1} \left({v_{J+1}}^T P_J v_{J+1} + 1 \right)^{-1} \left(y_{J+1} - {v_{J+1}}^T h_J \right) \tag{16}$$

Since in Eq. (16) ${v_{J+1}}^T P_J v_{J+1} + 1$ is scalar, matrix inversion is not required to update the value of h [15]. Eq. (16) states that updated value of h is the former h plus a weighted error between y_{J+1} and y_J [15]. This article aims to investigate the response of given methodology in computation of magnitude of symmetrical components.

The simulated results are provided in table 2. Table 3 explains the validity of proposed methodology.

5. Conclusion

Digital protection system is the need of modern era. Microprocessor based protection system has gained great importance since past few decades. Simplicity of algorithm plays an important role in the complexity of power system protection design. A simple reliable algorithm based on least squares has been investigated and analyzed. The given algorithm computes magnitude of symmetrical components by adding impact of newly arrived sample instead of calculating result again, makes it simple and suitable implementation in microprocessors. The fast determination of symmetrical components can lead us less power outage. However, the vulnerability of proposed algorithm can more be reduced by using high performance processors.

The tabulated result explains the validity of proposed method under all normal and faulty conditions of power system.

Table 3. Comparison of simulation results and historical data

Conditions	Simulated Results of Figure 8			Historical Results [8]		
	Negative sequence	Positive sequence	Zero sequence	Negative sequence	Positive sequence	Zero sequence
Normal	Almost 0	Only Positive Exists	Almost 0	0	Only Positive Exists	0
SLG	Negative and Zero are Almost same	Value of Positive is greater than other two	Negative and Zero are Almost same	Negative and Zero are same	Value of Positive is greater than other two	Negative and Zero are same
DLG	All three are Almost equal	All three are Almost equal	All three are Almost equal	All three are equal	All three are equal	All three are equal
L-L	Positive and Negative are both equal	Positive and Negative are both equal	Almost 0	Positive and Negative are both equal	Positive and Negative are both equal	0
L-L-L	Almost Zero	Almost Zero	Almost Zero	0	0	0
1-Conductor Broken	Negative and Zero are Almost same	Value of Positive is greater than other two	Negative and Zero are Almost same	Negative and Zero are same	Value of Positive is greater than other two	Negative and Zero are same
2-Conductor Broken	All three are Almost equal	All three are Almost equal	All three are Almost equal	All three are equal	All three are equal	All three are equal

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Appendix

The prove of least square method can easily be made using simple algebraic rules, consider a set of linear equations expressed as

$$y^o = hx$$

$y^o, h\ and\ x$ denotes output, input and weighting factor respectively. The weighting factor is calculated providing minimum error between desired output and estimated output which results as

$$e(x) = \left\|y - hx\right\|_2^2$$

Here $e(x)$ is the error between estimated output and desired output. Expansion of above equation yields

$$e(x) = (y - hx)^T(y - hx)$$
$$e(x) = y^T y - y^T hx - x^T h^T y + x^T h^T hx$$
$$e(x) = y^T y - 2y^T hx + x^T h^T hx$$
$$e(x) = y^T y - 2y^T hx + x^T h^T hx$$

Since $x^T h^T y$ and $y^T hx$ are scalar and transpose of each other. The derivative of $e(x)$ results

$$\frac{\partial e(x)}{\partial x} = -2h^T y + 2h^T hx$$

Setting derivative equal to zero

$$0 = -2h^T y + 2h^T hx$$
$$h^T hx = h^T y$$
$$x = (h^T h)^{-1} h^T y$$

And according to our scenario

$$h = (v^T v)^{-1} v^T y$$

The above value of weighting factor h is estimated on existing set of measurements. For any new measurement yields [15]

$$h_{J+1} = \left([v^T \vdots v_{J+1}][v \therefore v_{J+1}{}^T]\right)^{-1}[v^T \vdots v_{J+1}][y \therefore y_{J+1}]$$

The above equation involves continuous matrix inversion which is not satisfactory.

Let

$$P_J = (v^T v)^{-1}$$

For newly arrived sample

$$P_{J+1} = [v^T v + v_{J+1} {v_{J+1}}^T]^{-1}$$
$$P_{J+1} = [{P_J}^{-1} + v_{J+1} {v_{J+1}}^T]^{-1}$$

Since Henderson and Searle identity of matrixes is

$$(A + B)^{-1} = A^{-1} - A^{-1}B(I + A^{-1}B)^{-1}A^{-1}$$

Applying to P_{J+1}

$$P_{J+1} = P_J - P_J v_{J+1} {v_{J+1}}^T \left(I + P_J v_{J+1} {v_{J+1}}^T\right)^{-1} P_J$$
$$P_{J+1} = P_J - P_J v_{J+1} \left(I + {v_{J+1}}^T P_J v_{J+1}\right)^{-1} {v_{J+1}}^T P_J$$

h_{J+1} can further simplified as

$$h_{J+1} = P_{J+1}[v^T y + v_{J+1} y_{J+1}]$$
$$h_{J+1} = h_J + P_J v_{J+1} \left({v_{J+1}}^T P_J v_{J+1} + 1\right)^{-1} \left(y_{J+1} - {v_{J+1}}^T h_J\right)$$

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