

An Optimized PMSG Model for FPGA Hardware Development

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Abstract—With the increasing application of wind power generation, the need for transient modeling of wind turbines is becoming more and more urgent. In this paper, a PMSG high-order electromagnetic transient model that can be applied to discrete small-step simulation is established and optimized. The calculation of the high-order transient PMSG model is very complicated, and it is difficult to build a PMSG model on the hardware development platform for calculation. Therefore, this paper optimizes the calculation process of the PMSG model and decouples the traditional matrix inversion calculation into a linear equation calculation. After simulation and hardware implementation, it is proved that the simplified algorithm has good performance in convergence performance and calculation accuracy. Although the simplified algorithm oscillates at a higher frequency, it has a fast convergence speed, and the convergence result is the same as the exact calculation without approximation. The simplified algorithm is developed on the FPGA hardware platform. The hardware experiment proves that the algorithm occupies very few board resources and the board power consumption is very low.

Keywords—PMSG; high-order transient model; model optimization; FPGA;

I. INTRODUCTION

As an effective means to solve the energy crisis and environmental pollution, wind power generation is developing rapidly around the world [1-2]. Since the structure and operating characteristics of wind turbines are different from those of traditional synchronous generators, a large number of wind turbines connected to the grid will have a new impact on the transient stability of the power system [3]. To study the transient stability of a power system with wind power access, it is first necessary to model the wind turbine. Among the commonly used wind turbines, the PMSG (direct-driven permanent magnet synchronous generator) has the advantages of high operational reliability, low maintenance cost, and strong reactive power regulation capability. The proportion of PMSG in grid-connected wind turbines is gradually increasing [4]. Therefore, the modeling problem of PMSG suitable for transient stability analysis has received more and more attention.

Some literatures have studied the modeling of PMSG, but it is relatively simple. For example, literature [5]

established a PMSG mathematical model including a wind turbine model, transmission system model, and generator model proposed the control strategy of pitch angle and generator speed, but does not consider the influence of the grid-side inverter. References [6-9] use different control strategies to study the PMSG system connected to the grid via frequency converters, achieve maximum wind energy tracking control, and decoupling control of grid-connected active and reactive power. Reference [10] established a PMSG simulation model based on MATLAB/SIMULINK software. Reference [11] studies a neutral point clamp inverter system for PMSG grid-connected, and proposes the corresponding control strategy of the inverter. Reference [12] emphatically analyzed the control problem of the double pulse width modulation (PWM) PMSG generator-side inverter and proposed to increase the constraint equation to determine the stable control scheme of the generator terminal voltage. This literature basically focuses on the research of wind turbines or frequency converters used in wind turbines. The research on wind turbine models is not deep enough, and the generator models used are mostly low-order models, which cannot meet the accuracy requirements of generator transient simulation.

In this paper, a high-order electromagnetic transient model of PMSG is built, and the model is extended to an electromagnetic transient program (EMTP) type solution. The discretized model can be applied to hardware-in-the-loop simulation and discrete small-step transient simulation platforms. And a calculation method of pre-step substitution is proposed. The simulation proves that the proposed pre-step replacement discretization model has high accuracy, and can greatly reduce the hardware resource occupation and the time required for each step calculation

II. TRADITIONAL PMSG MODEL

To establish the mathematical model of the PMSG, the necessary assumptions must be made on the actual three-phase synchronous machine to simplify the analysis and calculation. It is usually assumed that:

- The magnetic permeability of the motor magnet part is constant.

- For the longitudinal axis and the transverse axis, the motor rotor is completely symmetrical in structure.
- The positions of the three windings of the stator differ from each other by an electrical angle of 120°, which are identical in structure and produce a sinusoidally distributed magnetomotive force.
- The stator and rotor of the motor have smooth surfaces.

A machine that satisfies the above assumptions is called an ideal machine. These assumptions can meet the needs of practical engineering research in most cases.

The dynamics of a PMSG can be represented by equations of corresponding electrical and mechanical subsystems. Without loss of generality, this paper considers a three-phase PMSG with a damping winding kd on the d-axis and a damping winding kq on the q-axis. The q-axis of the rotor reference frame is assumed to be 90 ahead of the d-axis. Assuming that the equations of the mechanical subsystem are the same for all models, the formula is as follows:

$$\begin{aligned} p\theta_r &= \omega_r \\ p\omega_r &= \frac{P}{J}(T_m - T_e) \end{aligned} \quad (1)$$

p is the differential operator, $p=d/dt$. θ_r and ω_r are the rotor position and the mechanical angular velocity; T_m and T_e are the mechanical torque and electromagnetic torque.

A. Traditional dq Axis Model

The equation of the motor model can be obtained by the Park transformation to obtain the dq axis voltage equation. The voltage equation includes stator and rotor and can be compactly expressed as:

$$\mathbf{v}_{dq0} = \mathbf{R}\mathbf{i}_{dq0} + p\boldsymbol{\lambda}_{dq0} + \mathbf{S} \quad (2)$$

The first term to the right of the equal sign is the ohmic voltage term, which reflects the resistive voltage drop across the corresponding winding.

The second term often referred to as the transformer electromotive force, is the winding voltage caused by electromagnetic induction effects.

The third term is called velocity electromotive force. This term is zero when the rotor is stationary ($\omega_r = 0$). The velocity electromotive force term reflects the electromotive force caused by the stator winding cutting the magnetic field lines, and it plays a major role in the energy exchange between the stator and the rotor.

$$\mathbf{v}_{dq0} = \begin{bmatrix} v_d & v_q & v_0 & 0 & 0 \end{bmatrix}^T \quad (3)$$

$$\mathbf{i}_{dq0} = \begin{bmatrix} i_d & i_q & i_0 & i_{kdr} & i_{kqr} \end{bmatrix}^T \quad (4)$$

$$\boldsymbol{\lambda}_{dq0} = \begin{bmatrix} \lambda_d & \lambda_q & \lambda_0 & \lambda_{kdr} & \lambda_{kqr} \end{bmatrix}^T \quad (5)$$

$$\mathbf{R} = \text{diag}(r_s \quad r_s \quad r_s \quad r_{kdr} \quad r_{kqr}) \quad (6)$$

$$\mathbf{S} = \begin{bmatrix} -\lambda_q \omega_r & \lambda_d \omega_r & 0 & 0 & 0 \end{bmatrix}^T \quad (7)$$

The flux linkage equations may be expressed as:

$$\begin{bmatrix} \lambda_d \\ \lambda_{kdr} \end{bmatrix} = \begin{bmatrix} L_d & L_{md} \\ L_{md} & L_{kdr} \end{bmatrix} \begin{bmatrix} i_d \\ i_{kdr} \end{bmatrix} + \begin{bmatrix} \lambda_{mr} \\ \lambda_{mr} \end{bmatrix} \quad (8)$$

$$\begin{bmatrix} \lambda_q \\ \lambda_{kqr} \end{bmatrix} = \begin{bmatrix} L_q & L_{mq} \\ L_{mq} & L_{kqr} \end{bmatrix} \begin{bmatrix} i_q \\ i_{kqr} \end{bmatrix} \quad (9)$$

$$\lambda_0 = L_{ls} i_0 \quad (10)$$

Finally, the electromagnetic torque is expressed as follows:

$$T_e = \frac{3P}{2} (\lambda_d i_q - \lambda_q i_d) \quad (11)$$

The meanings of the above parameters are shown in the table below:

TABLE I. MEANING OF PARAMETERS

Parameter	Meaning of Parameters
P	Generator Pole Pairs
J	Angular Moment of Inertia
Rs	Stator Winding Resistance
Lls	Stator Leakage Reactance
Ld	D-Axis Unsaturated Reactance
Lq	Q-Axis Unsaturated Reactance
Rkd	D-Axis Damper Winding Resistance
Lkd	D-Axis Damper Winding Reactance
Rkq	Q-Axis Damper Winding Resistance
Lkq	Q-Axis Damper Winding Reactance
Lmd	D-Axis Magnetizing Inductances
Lmq	Q-Axis Magnetizing Inductances

B. Discrete-Time Model

The implicit trapezoidal rule is one of the most basic numerical calculation methods in the transient analysis of power systems. It has good calculation accuracy and numerical stability and is a relatively reliable method. The implicit trapezoidal rule is as follows:

$$i(t) = \int_0^t A(t) dt = \frac{\Delta t}{2} \times [A(t) + A(t - \Delta t)] + i(t - \Delta t) \quad (12)$$

In order to obtain numerical solutions of the PMSG model within the EMTP, the implicit trapezoidal rule is applied to obtain the corresponding differential equations. Discretizing (1), the difference equations for the rotor position and speed are obtained as:

$$\theta_r(t) = \frac{\Delta t}{2} \times [\omega_r(t) + \omega_r(t - \Delta t)] + \theta_r(t - \Delta t) \quad (13)$$

$$\omega_r(t) = \frac{P\Delta t}{2J} \left[T_m(t) + T_m(t - \Delta t) \right] - T_e(t) - T_e(t - \Delta t) + \omega_r(t - \Delta t) \quad (14)$$

$$T_e(t) = \frac{3P}{2} [\lambda_d(t) i_q(t) - \lambda_q(t) i_d(t)] \quad (15)$$

Expressions for the derivatives of the stator and rotor current are obtained by manipulating the flux linkage equations (8) and (9). Substituting the currents expressions into (2), after algebraic manipulation, an exact form of the stator and rotor current equations is obtained:

$$p i_d = \frac{L_{kdr}}{L_d L_{kdr} - L_{md}^2} \left(+v_d - r_s i_d + L_q \omega_r i_q + \frac{L_{md} r_{kdr}}{L_{kdr}} i_{kdr} + L_{mq} \omega_r i_{kqr} \right) \quad (16)$$

$$p i_q = \frac{L_{kqr}}{L_q L_{kqr} - L_{mq}^2} \left(+v_q - L_d \omega_r i_d - L_{md} \omega_r i_{kdr} + \frac{L_{mq} r_{kqr}}{L_{kqr}} i_{kqr} - \lambda_{mr} \omega_r - r_s i_q \right) \quad (17)$$

$$p i'_{kd} = \frac{L_{md}}{L_{md}^2 - L_d L_{kdr}} \left(+v_d - r_s i_d + L_q \omega_r i_q + \frac{L_d r_{kdr}}{L_{md}} i_{kdr} + L_{mq} \omega_r i_{kqr} \right) \quad (18)$$

$$p i'_{kq} = \frac{L_{mq}}{L_{mq}^2 - L_q L_{kqr}} \left(+v_q - L_d \omega_r i_d - L_{md} \omega_r i_{kdr} + \frac{L_q r_{kqr}}{L_{mq}} i_{kqr} - \lambda_{mr} \omega_r - r_s i_q \right) \quad (19)$$

Considering the space in the article, the formula after algebraic manipulation is not shown here. The final PMSG high-order transient model can be organized into the following form:

$$p \mathbf{i}(t) = \mathbf{A} \mathbf{i}(t) + \mathbf{B} \mathbf{v}(t) \quad (20)$$

$$\mathbf{i}(t) = \begin{bmatrix} i_d & i_q & i_{kdr} & i_{kqr} \end{bmatrix}^T \quad (21)$$

$$\mathbf{v}(t) = \begin{bmatrix} v_d & v_q & \lambda_{mr} \end{bmatrix}^T \quad (22)$$

Matrix A is a parameter matrix with 4 rows and 4 columns, where the parameters change as the rotor mechanical angular velocity changes. Matrix B is a sparse parameter matrix with 4 rows and 3 columns, and the parameters are also affected by the mechanical angular velocity of the rotor.

Apply the implicit trapezoidal rule to the equation (20).

$$\mathbf{i}(t) = \frac{\Delta t}{2} \left[\begin{matrix} +\mathbf{A} \mathbf{i}(t) + \mathbf{A} \mathbf{i}(t - \Delta t) \\ +\mathbf{B} \mathbf{v}(t) + \mathbf{B} \mathbf{v}(t - \Delta t) \end{matrix} \right] + \mathbf{i}(t - \Delta t) \quad (23)$$

$$\mathbf{i}(t) = \left[\mathbf{E} - \frac{\Delta t}{2} \mathbf{A} \right]^{-1} \times \frac{\Delta t}{2} \left[\begin{matrix} \mathbf{A} \mathbf{i}(t - \Delta t) + \mathbf{B} \mathbf{v}(t) \\ +\mathbf{B} \mathbf{v}(t - \Delta t) \end{matrix} \right] + \left[\mathbf{E} - \frac{\Delta t}{2} \mathbf{A} \right]^{-1} \times \mathbf{i}(t - \Delta t) \quad (24)$$

where E is the identity matrix. This method can be used to calculate the PMSG model current, but it is clearly shown in the formula that in the hardware-based small step discretization calculation, each step needs to calculate an inverse matrix, which will occupy a lot of board computing resources and affect the calculating speed. Especially when the transient simulation of the wind farm is carried out with hardware, the calculation amount of the matrix inversion will be very large.

III. SIMPLIFIED CALCULATION METHOD

On this basis, this paper focuses on how to reduce the calculation amount of PMSG model operation, in order to achieve the purpose of reducing board resource occupancy and improving calculation speed. In the small-step simulation, the data difference between each step is very small, so the actual hardware calculation can be replaced by the data of the previous step, and the matrix inversion problem is decoupled into a linear equation calculation problem.

The matrix $\mathbf{A} \mathbf{i}(t)$ is arranged as follows, a represents the specific parameters of each item in the matrix

$$\mathbf{A} \mathbf{i}(t) = \begin{bmatrix} a_{11}(t) & \cdots & a_{14}(t) \\ \vdots & \ddots & \vdots \\ a_{41}(t) & \cdots & a_{44}(t) \end{bmatrix} \quad (25)$$

The upper triangular parameters of the matrix are replaced by the parameters of the previous step for calculation. The matrix can be obtained as follows:

$$\mathbf{A} \mathbf{i}(t) = \begin{bmatrix} a_{11}(t) & \cdots & a_{14}(t - \Delta t) \\ \vdots & \ddots & \vdots \\ a_{41}(t) & \cdots & a_{44}(t) \end{bmatrix} \quad (26)$$

In this way, the matrix inversion problem is decoupled into a linear system computation problem.

IV. COMPUTER STUDIES

A single-generator, resistive inductive load system is assumed here to compare the different models. Resistive and inductive loads are used to simulate residential electricity consumption in real life.

The machine parameters obtained from [13] are summarized in the table below.

TABLE II. MACHINE PARAMETERS

Parameter	Data	Parameter	Data
Rs	7.955 Ω	Lls	0.508 mH
Ld	7.681 mH	Lq	15.502 mH
Rkd	0.257 Ω	Lkd	8.659 mH
Rkq	0.856 Ω	Lkq	16.410 mH
Lmd	7.173 mH	Lmq	14.994 mH

PMSG parameters [13]: 3.419 MVA, 4 kV, 0.716 pf, 8 poles, 400 r/min, $J = 4$ MWs.

The d-axis voltage v_d obtained from the simulation is shown in the following figure, in which Fig.1 is the waveform diagram of the generator running in a transient state, and Fig.2 is the waveform diagram after the generator is in a stable state. The pink solid line in the figure is the simplified linear algorithm waveform, and the black dotted line is the exact matrix inversion algorithm

waveform.

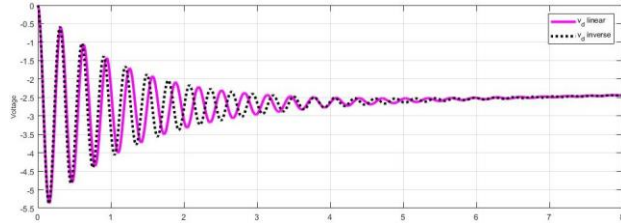


Fig. 1. v_d waveform in transient state.

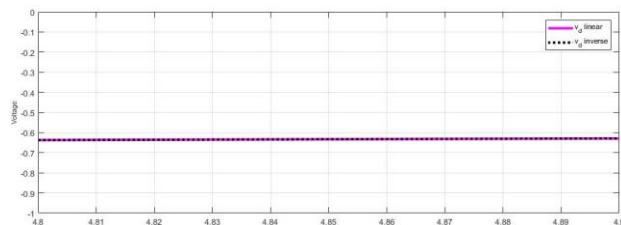


Fig. 2. v_d waveform in stable state.

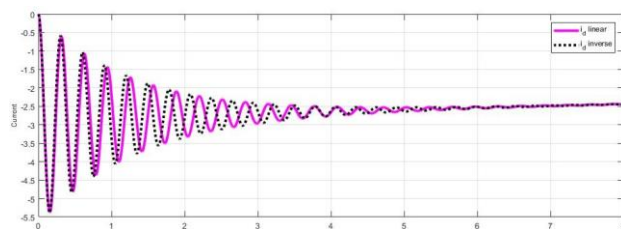


Fig. 3. i_d waveform in transient state.

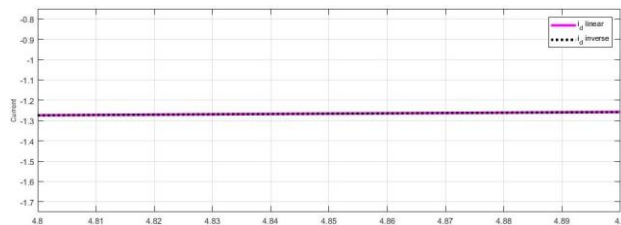


Fig. 4. i_d waveform in stable state.

It can be seen from the above figure that when the generator is running in a transient state at the beginning of the operation, the two algorithms have good convergence. The simplified algorithm has more oscillations and higher oscillation frequency. However, the time for the two methods to finally converge to stability is similar, and the final convergence results are similar too.

FPGA (Field Programmable Gate Array) is a chip with powerful computing power. It has a highly parallel hardware structure, distributed memory blocks, programmable features, and rich I/O resources. It is the first choice for real-time transient simulation.

The simplified algorithm model is built on the FPGA supporting development platform Vivado.

In this experiment, Xilinx Virtex®-7 FPGA VC707 was selected as shown in the figure below:

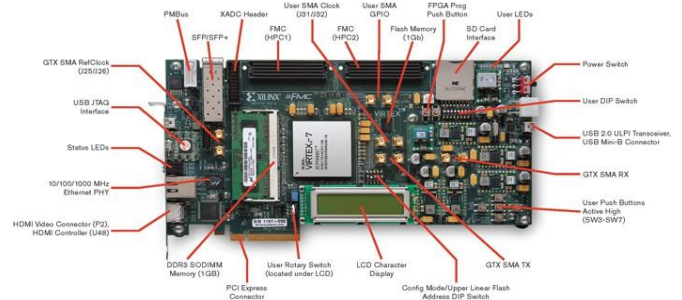


Fig. 5. Xilinx Virtex®-7 FPGA VC707

The Virtex®-7 FPGA VC707 Evaluation Kit is a full-featured, highly flexible, high-speed serial base platform that includes basic components of hardware, design tools, IP, and pre-verified reference designs for system designs that demand high-performance, serial connectivity, and advanced memory interfacing.

The RTL (Real Time Logistics) diagram for the optimized PMSG model is shown below:

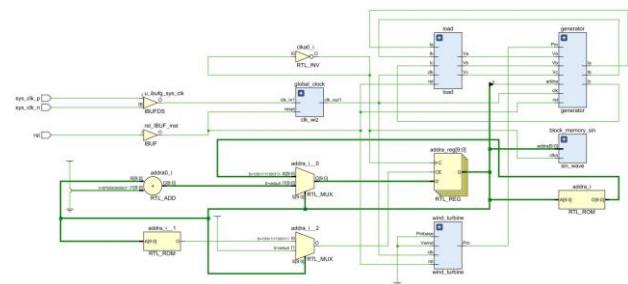


Fig. 6. RTL diagram.

After the synthesis, implementation, and generate bitstream processes of the hardware development platform, the resource occupancy of the FPGA and the power consumption temperature of the FPGA is shown in the figure below.

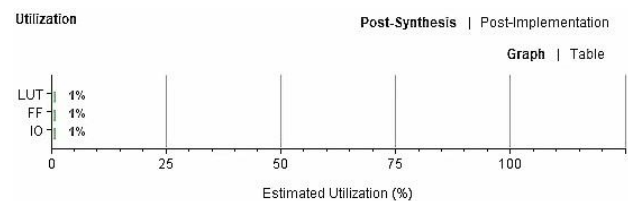


Fig. 7. FPGA estimated utilization.

Power		Summary	On-Chip
Total On-Chip Power:	0.24 W		
Junction Temperature:	25.3 °C		
Thermal Margin:	59.7 °C (50.6 W)		
Effective θ_{JA} :	1.1 °C/W		
Power supplied to off-chip devices:	0 W		
Confidence level:	High		

Fig. 8. FPGA power consumption and temperature.

As can be seen from the above figure, the simplified algorithm only takes up very few board resources, the overall resource occupation is less than 5%, and the overall power consumption of the board is only 0.24w, which is very resource-saving.

V. CONCLUSION

The high-order transient PMSG model built in the transient simulation has a large number of calculations, and the calculation is very complicated. It is difficult to build a PMSG model on the hardware development platform for calculation. Therefore, this paper optimizes the calculation process of the PMSG model and decouples the traditional matrix inversion calculation into a linear equation system calculation. After simulation and hardware implementation, it is proved that the simplified algorithm has good performance in convergence performance and calculation accuracy. Although the simplified algorithm oscillates more and the oscillation frequency is higher, it has a fast convergence speed, and the convergence result is consistent with the accurate calculation without approximation. The simplified algorithm is developed on the FPGA hardware platform. The hardware experiment proves that the algorithm occupies very few board resources and the board power consumption is very low.

REFERENCES

- [1] J. X. Cao, G. J. Wang, L. I. Jian-Min, "Researches on Dynamic Performances of Reactive Power Compensation in Wind Power Grid System," *Power Capacitor & Reactive Power Compensation*, 2012, 33(3): 16-22, 60.
- [2] J. B. Song, W. X. Pan, P. U. Zhen, B. Wang, H. Tian, "Optimization Design of Wind Turbine Grounding Grid Considering Touch Voltage," *Insulators and Surge Arresters*, 2013, (6): 56-60.
- [3] G. F. Fan, H. X. Zhao, H. Z. Dai, "The Impact and Countermeasure of Large Scale Wind Power on Power System," *Power System and Clean Energy*, 2008, 24(1): 44-48.
- [4] X. Zhu, C. Qin, X. Qiu, S. Luo, J. Qin, "Research on Energy Storage Capacity Allocation for Low Voltage Operation of the Permanent Magnetic Synchronous Generator," *Power System & Clean Energy*, 2015, 31(3): 106-112.
- [5] M. Yin, L. I. Geng-Yin, J. C. Zhang, W. R. Zhao, Y. F. Xue, "Modeling and Control Strategies of Directly Driven Wind Turbine with Permanent Magnet Synchronous Generator," *Power System Technology*, 2007, 31(15): 61-65.
- [6] K. Xu, H. Min-qiang, Z. Jian-yong, "Direct controlling of the grid-side power for the wind power generator without speed sensor," *Automation of Electric Power Systems*, 2006, 30(23): 43-47.
- [7] Q. Liu, Y. He, R. Zhao, "The maximal wind-energy tracing control of avariable-speed constant-frequency wind-power generation system," *Automation of Electric Power Systems*, 2003, 27 (20): 62- 67.
- [8] R. Esmaili, L. Xu, D. K. Nichols, "A new control method of permanent magnet generator for maximum power tracking in wind turbine application," *Power Engineering Society General Meeting IEEE*, 2005. 3(6): 12- 16.
- [9] Y. M. Yan, Y. Fan, Z. Z. Wang, "Self-optimization control of PM synchronous wind turbine generator system," *Transactions of China Electrotechnical Society*, 2002, 17(6): 82- 86.
- [10] D. Svehkarenko, "Simulations and Control of Direct Driven Permanent Magnet Synchronous Generator," 2005.
- [11] A. Yazdani, R. Iravani, "A neutral-point clamped converter system for direct-drive variable-speed wind power unit," *IEEE Transactions on Energy Conversion*, 2006, 21(2):596-607.
- [12] Z. F. Feng, E.X. Yang, G. Z. Chen, "Unity power factor control straight of direct driven permanent magnet synchronous generator based wind turbine," *Mechanical and Electrical Engineering Magazine*, 2007,24(9): 70- 73.
- [13] B. Wu, Y. Lang, N. Zargari, S. Kouro, "Power Conversion and Control of Wind Energy Systems," *IEEE Press, Wiley*, 2011.