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Steady-state power operation region of a modular multilevel converter connecting to an AC grid

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Abstract

The modular multilevel converter (MMC) has become a widely used topology for voltage source converter-based high-voltage direct current transmission projects. To calculate the power operation region of an MMC directly and efficiently, we propose a novel steady-state phasor model of the MMC. The model is expressed as an equivalent capacitance in series with a voltage source, where parameters are independent of alternating current (AC) electrical quantity. Based on the model, we propose an open-loop approach to determine the operation region boundary of MMC by calculating the envelopes of the P–Q curves. The effect of MMC parameters, the circulating current control, and the short-circuit ratio of the AC grid to the power operation region can be determined by the size and position of the power operation region. The proposed approach and analysis results were verified by an MMC/AC simulation model built in PSCAD/EMTDC.

1 | INTRODUCTION

High-voltage direct current (HVDC) transmission systems and technologies continue to advance as they make their way to commercial applications [1–3]. It was based initially on thyristor technology and, more recently, on fully controlled semiconductors and voltage-source converter (VSC) topologies [3]. The modular multilevel converter (MMC) is suitable for high-voltage and large-capacity power transmission due to its modular structure and has become a widely used topology for VSC–HVDC transmission projects [4–6]. The MMC and the two-level VSC have apparent differences in steady-state operating characteristics, where all the electrical quantities and control variables are coupled in an MMC due to the effects of capacitor voltage ripples in each submodule (SM) [7, 8].

Studying the operation region of an MMC plays an essential role in the parameter design of an MMC–HVDC system. The VSC's active and reactive power output is determined by its virtual electromotive force (EMF). Reference [9] determined the operation region of the MMC by borrowing

from the two-level VSC model and mapping the MMC's power outputs to its EMF. This method can determine the boundary of the operation region. However, there is a specific deviation in the calculation result without considering the difference between MMC and two-level VSC. So far, most current methods to determine the operation region of MMC are 'seeking-and-checking' or 'point-scanning' analysis methods instead of determining the boundary of the operation region. In [10], the constraints of the voltage stability, the voltage, and the current are studied. Reference [11] examined the limitations of internal dynamics. In [12], researchers studied the operation region of an MMC under unbalanced power grid conditions. The influence of circulating current control on the power operation region is studied in [13, 14]; however, the methods to determine the operation region are proposed for the circulating current suppression and injection controls, not universal. Hence, a general approach to determine the operating region boundary of the MMC directly is still needed.

The key to accurately determine the operation region boundary of MMC is to establish a steady-state MMC with

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an open-loop solution method. Much research has been conducted on the steady-state model of MMC. A steady-state time-domain analysis model of an MMC was proposed in [15], ignoring the circulating current and its suppression control. Reference [16] proposed a simplified phasor model of the MMC, ignoring the circulating current. The accurate models presented in [8, 17] can solve the circulating current, where the accuracy is relatively high. In [8], a closed-loop time-domain analysis model of MMC was established by defining an accurate switching function. The authors of [17] proposed a general MMC steady-state analysis model for MMC's optimization considering various circulating current control methods [13, 18–22]; that is, the circulating current injection control, the third-harmonic signal injection control, and the average capacitor voltage control. The existing steady-state models can accurately describe the coupling characteristics of an MMC. Still, an iterative solution had to be adopted because they failed to decouple the control variables and electrical quantities. There is a problem in and around the critical point that the iteration may not converge.

In this paper, we study the open-loop method to determine the power operation region boundary of an MMC. We first proposed a novel steady-state phasor model of MMC by deriving the function relation between the output voltage and current of MMC. Based on the model, determining the steady-state power operation region is transformed to solve the envelopes of a family of P–Q curves. The rest of this paper is organised as follows: In Section 2, the time-domain model of MMC is described to state the principles and assumptions of the MMC steady-state model. In Section 3, the complete form of the novel steady-state phasor MMC model is proposed, and its derivation is presented in detail. Based on the proposed model, a method to determine the steady-state power operation region of an MMC is proposed in Section 4. Section 5 describes the simulation validation in PSCAD/EMTDC that was done to verify the research in this study. In Section 6, the conclusions are summarised.

2 | TIME-DOMAIN MODEL OF MMC

2.1 | Equivalent circuit and mathematical model of MMC

The single-phase equivalent circuit of an MMC [8] is shown in Figure 1. In each arm, there are N series-connected SMs and an arm inductor L_0 . The arm voltages across the N series-connected SMs in the upper and lower arms of phase i are u_{pi} and u_{ni} , respectively, and the arm currents are i_{pi} and i_{ni} , respectively. The MMC is connected to the alternating current (AC) grid at the point of common coupling (PCC) through a three-phase converter, where L_T is the leakage reactance, u_i and u_{vi} are the single-phase AC voltage on the grid side and valve side, respectively, and i_{ti} and i_{vi} are the single-phase AC currents on the grid side and valve side, respectively. Figure 1b shows the equivalent model of an MMC from the AC side,

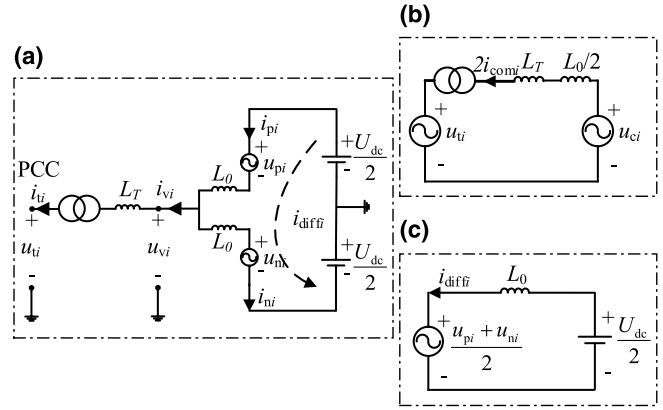


FIGURE 1 MMC single-phase model. (a) Single-phase equivalent model, (b) AC equivalent model, (c) DC equivalent model. AC, alternating current; DC, direct current; PCC, point of common coupling; MCC, modular multilevel converter

including u_{ci} , the inner EMF of phase i , which is also called the output voltage of the MMC, and i_{ci} , which is the output current of the MMC. Figure 1c shows the equivalent model of the MMC from the direct current (DC) side, where U_{dc} is the DC line voltage. u_{diff} and u_{com} , defined by Equation (1), are the differential-mode and common-mode components of the arm voltages, respectively. The definitions of i_{diff} and i_{com} , the differential-mode and common-mode components of arm current, are shown by Equation (2).

$$\begin{cases} u_{comi} = -\frac{u_{pi} - u_{ni}}{2} = u_{ci} \\ u_{diffi} = \frac{u_{pi} + u_{ni}}{2} = \frac{U_{dc}}{2} - u_{L0} \end{cases} \quad (i = a, b, c), \quad (1)$$

where u_{L0} is the voltage across the arm inductor.

$$\begin{cases} i_{comi} = \frac{i_{pi} - i_{ni}}{2} = \frac{1}{2}i_{vi} \\ i_{diffi} = \frac{i_{pi} + i_{ni}}{2} = \frac{1}{3}i_{dc} + i_{ciri} \end{cases} \quad (i = a, b, c), \quad (2)$$

where i_{dc} is the DC current and i_{ciri} is the circulating current.

Assuming capacitor voltages are well balanced, and the switching frequency is very high, all SMs in one arm can be considered identical [8]. Therefore, all capacitors in the N series SMs of one arm can be equivalent to an equivalent arm capacitor C_{eq} , where $C_{eq} = C_0/N$, and C_0 is the SM capacitance. The current flow across the equivalent arm capacitor is produced by the corresponding switching function and arm current. By integrating it, the corresponding voltage of the equivalent arm capacitor is obtained. The upper and lower arm voltages are produced by multiplying the corresponding equivalent arm capacitor's voltage with the switching function. The mathematical expression can be written as Equation (3).

$$\begin{cases} i_{\text{cap_pi}} = s_{\text{pi}} i_{\text{pi}} \\ i_{\text{cap_ni}} = s_{\text{ni}} i_{\text{ni}} \\ u_{\text{cap_pi}}^{\Sigma} = u_{\text{cap0}}^{\Sigma} + \frac{1}{C_{\text{eq}}} \int i_{\text{cap_pi}} dt \\ u_{\text{cap_ni}}^{\Sigma} = u_{\text{cap0}}^{\Sigma} + \frac{1}{C_{\text{eq}}} \int i_{\text{cap_ni}} dt \\ u_{\text{pi}} = s_{\text{pi}} u_{\text{cap_pi}}^{\Sigma} \\ u_{\text{ni}} = s_{\text{ni}} u_{\text{cap_ni}}^{\Sigma} \end{cases} \quad (i = a, b, c), \quad (3)$$

where s_{pi} and s_{ni} are the switching functions of the upper and lower arms of phase i , respectively; $i_{\text{cap_p}}$ and $i_{\text{cap_n}}$ are the capacitor currents of the upper and lower arms, respectively; $U_{\Sigma \text{ cap_pi}}$ and $U_{\Sigma \text{ cap_ni}}$ are the equivalent arm capacitor voltages of the upper and lower arms, respectively; and $U_{\Sigma \text{ cap0}}$ is the DC component of the equivalent arm capacitor voltage.

2.2 | Steady-state equivalent and coordinate transformation

In steady-state, ignoring the third- and higher-order components, the arm current of the upper and lower arms of phase i can be written as Equation (4), and the accurate switching functions [8] of the upper and lower arms can be defined as Equation (5). In [8], by substituting Equations (4) and (5) into Equations (1)–(3), the steady-state time-domain model of an MMC was obtained.

$$\begin{cases} i_{\text{pi}} = I_{\text{diff0}} + I_{\text{com}} \cos(\omega t - \varphi_{\text{comi}}) + I_{\text{diff2}} \cos(2\omega t - \varphi_{\text{diff2i}}) \\ i_{\text{ni}} = I_{\text{diff0}} - I_{\text{com}} \cos(\omega t - \varphi_{\text{comi}}) + I_{\text{diff2}} \cos(2\omega t - \varphi_{\text{diff2i}}) \end{cases} \quad (i = a, b, c) \quad (4)$$

where I_{diff0} is the DC component of the differential-mode arm current, I_{com} and φ_{comi} are the magnitude and angle of the fundamental component of the common-mode arm current of phase i , and I_{diff2} and φ_{diff2i} are the magnitude and angle of the second-order component of the differential-mode arm current.

$$\begin{cases} s_{\text{pi}} = \frac{1}{2} M_{\text{dc}} - \frac{1}{2} M_{\text{e}} \cos(\omega t + \theta_{\text{ei}}) + \frac{1}{2} M_2 \cos(2\omega t + \theta_{2i}) \\ s_{\text{ni}} = \frac{1}{2} M_{\text{dc}} + \frac{1}{2} M_{\text{e}} \cos(\omega t + \theta_{\text{ei}}) + \frac{1}{2} M_2 \cos(2\omega t + \theta_{2i}) \end{cases} \quad (i = a, b, c) \quad (5)$$

where M_{dc} is the DC modulation index; M_{e} and θ_{ei} are the magnitude and the phase angle of the fundamental modulation

components, which come from the inner-loop current controller's outputs; and M_2 and θ_{2i} are the magnitude and the phase angle of the second-order modulation components, which come from the circulating current controller's outputs.

The Park transformation matrix from the abc reference frame to the $dq0$ rotating frame is defined as Equation (6), where the superscript $+$ represents the positive sequence, and the superscript $-$ represents the negative sequence.

$$\begin{cases} P^+ = \frac{2}{3} \begin{bmatrix} \cos \theta & \cos(\theta - 2\pi/3) & \cos(\theta + 2\pi/3) \\ -\sin \theta & -\sin(\theta - 2\pi/3) & -\sin(\theta + 2\pi/3) \\ 1/2 & 1/2 & 1/2 \end{bmatrix} \\ P^- = \frac{2}{3} \begin{bmatrix} \cos \theta & \cos(\theta + 2\pi/3) & \cos(\theta - 2\pi/3) \\ -\sin \theta & -\sin(\theta + 2\pi/3) & -\sin(\theta - 2\pi/3) \\ 1/2 & 1/2 & 1/2 \end{bmatrix} \end{cases} \quad (6)$$

Thus, Equations (4) and (5) can be rewritten in the form of d and q components, as shown by Equations (7) and (8), respectively. The subscripts d and q are the corresponding d and q components, respectively.

$$\begin{cases} i_{\text{pi}} = I_{\text{diff0}} + [I_{\text{comd}} \cos(\omega t + \theta_i^+) - I_{\text{comq}} \sin(\omega t + \theta_i^+)] \\ \quad + [I_{\text{diff2d}} \cos(2\omega t + \theta_i^-) - I_{\text{diff2q}} \sin(2\omega t + \theta_i^-)] \\ i_{\text{ni}} = I_{\text{diff0}} - [I_{\text{comd}} \cos(\omega t + \theta_i^+) - I_{\text{comq}} \sin(\omega t + \theta_i^+)] \\ \quad + [I_{\text{diff2d}} \cos(2\omega t + \theta_i^-) - I_{\text{diff2q}} \sin(2\omega t + \theta_i^-)] \end{cases} \quad (i = a, b, c) \quad (7)$$

$$\begin{cases} s_{\text{pi}} = \frac{1}{2} M_{\text{dc}} - \frac{1}{2} [M_{\text{ed}} \cos(\omega t + \theta_i^+) - M_{\text{eq}} \sin(\omega t + \theta_i^+)] \\ \quad + \frac{1}{2} [M_{2d} \cos(2\omega t + \theta_i^-) - M_{2q} \sin(2\omega t + \theta_i^-)] \\ s_{\text{na}} = \frac{1}{2} M_{\text{dc}} + \frac{1}{2} [M_{\text{ed}} \cos(\omega t + \theta_i^+) - M_{\text{eq}} \sin(\omega t + \theta_i^+)] \\ \quad + \frac{1}{2} [M_{2d} \cos(2\omega t + \theta_i^-) - M_{2q} \sin(2\omega t + \theta_i^-)] \end{cases} \quad (i = a, b, c) \quad (8)$$

where $\theta_+ a = 0$, $\theta_+ b = -2\pi/3$, $\theta_+ c = 2\pi/3$, $\theta_- a = 0$, $\theta_- b = 2\pi/3$, and $\theta_- c = -2\pi/3$; I_{comd} and I_{comq} are the d - and q -fundamental current components of arm currents; I_{diff2d} and I_{diff2q} are the d - and q -second-order components of arm

currents; M_{ed} and M_{eq} are the d - and q -fundamental modulation components; and M_{2d} and M_{2q} are the d - and q -second-order modulation components.

In a three-phase symmetrical condition, there are a total of five control components: M_{dc} , M_{ed} , M_{eq} , M_{2d} , and M_{2q} , which controls the five arm current components, including I_{diff0} , I_{comd} , I_{comq} , I_{diff2d} , and I_{diff2q} . Nevertheless, the control process is not decoupled due to the effects of capacitor voltage ripples in each submodule [7, 8].

3 | STEADY-STATE PHASOR MODEL OF MMC

3.1 | Derivation of MMC steady-state phasor model

To derive the function relationship between the MMC's output voltage U_c and the output current I_v in phasor form, let us define the current vector as $I_{MMC} = [I_{diff0} \ I_{comd} \ I_{comq} \ I_{diff2d} \ I_{diff2q}]^T$, the current vector of the equivalent capacitor as $I_{cap} = [I_{cap0} \ I_{capd} \ I_{capq} \ I_{cap2d} \ I_{cap2q}]^T$, the voltage vector of the equivalent capacitor as $U_{cap}^\Sigma = [U_{cap0}^\Sigma \ U_{capd}^\Sigma \ U_{capq}^\Sigma \ U_{cap2d}^\Sigma \ U_{cap2q}^\Sigma]^T$, and the output voltage vector as $U_{MMC} = [U_{diff0} \ U_{comd} \ U_{comq} \ U_{diff2d} \ U_{diff2q}]^T$. Define the switching function transformation matrix S_M , the equivalent capacitance voltage transformation matrix S_{Ucap} , and the voltage output transformation matrix S_{UMMC} , as shown by Equation (9). Thus, according to the symmetry of the upper and lower bridge arms, Equation (3) can be rewritten in the matrix form as shown by Equation (10).

$$\left\{ \begin{array}{l} S_M = \begin{bmatrix} \frac{M_{dc}}{2} & \frac{M_{ed}}{4} & \frac{M_{eq}}{4} & \frac{M_{2d}}{4} & \frac{M_{2q}}{4} \\ -\frac{M_{ed}}{2} & \frac{M_{dc}}{2} & -\frac{M_{2d}}{4} & \frac{M_{2q}}{4} & -\frac{M_{ed}}{4} \\ -\frac{M_{eq}}{2} & -\frac{M_{2q}}{4} & \frac{M_{dc}}{2} + \frac{M_{2d}}{4} & \frac{M_{eq}}{4} & -\frac{M_{ed}}{4} \\ -\frac{M_{2d}}{2} & \frac{M_{ed}}{4} & \frac{M_{eq}}{4} & \frac{M_{dc}}{2} & 0 \\ -\frac{M_{2q}}{2} & \frac{M_{eq}}{4} & \frac{M_{ed}}{4} & 0 & \frac{M_{dc}}{2} \end{bmatrix} \\ S_{Ucap} = \begin{bmatrix} 0 & 0 & 1/\omega C_{eq} & 0 & 0 \\ 0 & -1/\omega C_{eq} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/2\omega C_{eq} \\ 0 & 0 & 0 & -1/2\omega C_{eq} & 0 \end{bmatrix} \\ S_{UMMC} = \text{diag}([1 \ -1 \ -1 \ 1 \ 1])S_M \end{array} \right. \quad (9)$$

$$\left\{ \begin{array}{l} I_{cap} = S_M I_{MMC} \\ U_{cap}^\Sigma = [U_{cap0}^\Sigma \ S_{Ucap} I_{cap}]^T \\ U_{MMC} = S_{UMMC} U_{cap}^\Sigma \end{array} \right. \quad (10)$$

Note that the DC component of capacitance current equals 0 in the steady state, which is expressed as Equation (11):

$$I_{cap0} = \left[\frac{M_{dc}}{2} \quad -\frac{M_{ed}}{8} \quad -\frac{M_{eq}}{8} \quad \frac{M_{2d}}{2} \quad \frac{M_{2q}}{2} \right] I_{MMC} = 0. \quad (11)$$

According to Equation (1), the equivalent of the output voltage vector U_{MMC} is obtained as

$$U_{MMC} = [U_{dc}/2 \ U_{cd} \ U_{cq} \ -U_{L02d} \ -U_{L02q}]^T, \quad (12)$$

where
$$\begin{bmatrix} U_{L02d} \\ U_{L02q} \end{bmatrix} = \begin{bmatrix} 0 & -2\omega L_0 \\ 2\omega L_0 & 0 \end{bmatrix} \begin{bmatrix} I_{diff2d} \\ I_{diff2q} \end{bmatrix}$$

Substituting Equation (10) into Equations (11) and (12) and combining them, the steady-state operation equation of an MMC can be obtained as Equation (13). The expressions of the matrices in Equation (13) are shown by Equations (B-1)–(B-6) in Appendix B.

$$\left\{ \begin{array}{l} 0 = \mathbf{A} \begin{bmatrix} I_{comd} \\ I_{comq} \end{bmatrix} + \mathbf{B} \begin{bmatrix} U_{cap0}^\Sigma \\ I_{diff0} \\ I_{diff2d} \\ I_{diff2q} \end{bmatrix} + \mathbf{C} U_{dc} \\ \begin{bmatrix} U_{cd} \\ U_{cq} \end{bmatrix} = \mathbf{D} \begin{bmatrix} I_{comd} \\ I_{comq} \end{bmatrix} + \mathbf{E} \begin{bmatrix} U_{cap0}^\Sigma \\ I_{diff0} \\ I_{diff2d} \\ I_{diff2q} \end{bmatrix} + \mathbf{F} U_{dc} \end{array} \right. \quad (13)$$

Thus, the functional relationship between the AC voltage and current outputs of an MMC can be derived, as Equation (14) shows. It is the steady-state phasor model of MMC at the AC side.

$$\begin{bmatrix} U_{cd} \\ U_{cq} \end{bmatrix} = -\mathbf{Z}_{MMC} \begin{bmatrix} I_{vd} \\ I_{vq} \end{bmatrix} + \mathbf{E}_c, \quad (14)$$

$$\text{where } \begin{cases} \mathbf{Z}_{\text{MMC}} = -(\mathbf{D} - \mathbf{E}\mathbf{B}^{-1}\mathbf{A})/2 \\ \mathbf{E}_c = \mathbf{M}_{\text{MMC}}\mathbf{U}_{\text{dc}} = -(\mathbf{E}\mathbf{B}^{-1}\mathbf{C})\mathbf{U}_{\text{dc}} \end{cases}.$$

The impedance matrix $\mathbf{Z}_{\text{MMC}}$ can be written in the form of $\begin{bmatrix} 0 & -X_{\text{MMC}} \\ X_{\text{MMC}} & 0 \end{bmatrix}$, which means that the MMC from the AC side can be described as the equivalent model with the reactance X_{MMC} and the voltage source $\mathbf{E}_c$ in series.

Let us take $X_{L0} = \omega L_0$ and $X_{\text{Ceq}} = 1/(\omega C_{\text{eq}})$, and normalise them. The rated MMC DC voltage U_{dcN} and the rated DC power P_N are selected as the base values of the DC voltage and power, respectively. The selection method of U_{vN} , the rated voltage on the valve side of the converter transformer, is shown by Equation (15), and it is chosen as the base value of the MMC's AC output voltage.

$$U_{\text{vN}} = \frac{\sqrt{3}m_k U_{\text{dcN}}}{2\sqrt{2}}, \quad (15)$$

where m_k is the no-load transformation ratio of the converter transformer; usually 0.85.

Thus, the parameters of the equivalent model in a per-unit system will be obtained, as shown in Equation (16).

$$\begin{cases} X_{\text{MMC}} = -\frac{\zeta_1 + \zeta_2 + \zeta_3}{64(\zeta_1 + \zeta_2 + \zeta_3)} X_{\text{Ceq}} \\ \mathbf{E}_c = \frac{U_{\text{dc}}}{m_k M_{\text{dc}}} \left(1 - \frac{X_{\text{Ceq}} \left[(M_{\text{cd}}^2 - M_{\text{eq}}^2) M_{2d} + 2M_{\text{cd}} M_{\text{eq}} M_{2q} \right]}{(\zeta_1 + \zeta_2 + \zeta_3)} \right) \begin{bmatrix} M_{\text{cd}} \\ M_{\text{eq}} \end{bmatrix} \\ \quad + \frac{3X_{\text{Ceq}} U_{\text{dc}}}{m_k M_{\text{dc}} (\zeta_1 + \zeta_2 + \zeta_3)} \begin{bmatrix} M_{2d} & M_{2q} \\ M_{2q} & -M_{2d} \end{bmatrix} \begin{bmatrix} M_{\text{cd}} \\ M_{\text{eq}} \end{bmatrix} \end{cases} \quad (16)$$

where

$$\begin{cases} \zeta_1 = 32M_{\text{dc}}X_{L0}(8M_{\text{dc}}^2 - 3M_{\text{e}}^2 - 2M_2^2) \\ \zeta_2 = -M_{\text{dc}}X_{\text{Ceq}} \begin{pmatrix} 16M_{\text{dc}}^4 + 3M_{\text{e}}^4 + 2M_2^4 \\ -16M_{\text{dc}}^2 M_{\text{e}}^2 - 12M_{\text{dc}}^2 M_2^2 - 2M_{\text{e}}^2 M_2^2 \end{pmatrix}, \\ \zeta_3 = [32X_{L0} - (12M_{\text{dc}}^2 - 2M_{\text{e}}^2 + M_2^2)X_{\text{Ceq}}] \\ \quad [(M_{\text{cd}}^2 - M_{\text{eq}}^2)M_{2d} + 2M_{\text{cd}}M_{\text{eq}}M_{2q}] \\ \zeta_1 = 32M_{\text{dc}}X_{L0} \\ \zeta_2 = -M_{\text{dc}}[2M_{\text{dc}}^2 + M_{\text{e}}^2 - M_2^2]X_{\text{Ceq}} \\ \zeta_3 = [(M_{\text{cd}}^2 - M_{\text{eq}}^2)M_{2d} + 2M_{\text{cd}}M_{\text{eq}}M_{2q}]X_{\text{Ceq}} \end{cases},$$

where $M_{\text{e}}^2 = M_{\text{cd}}^2 + M_{\text{eq}}^2$ and $M_2^2 = M_{2d}^2 + M_{2q}^2$.

It should be noticed from Equation (14) that even when the DC voltage is specified, the EMF of the MMC $\mathbf{U}_c$ cannot be completely determined by the controller outputs, but is related to the AC currents. That is why [8] was required to adopt a closed-loop analysis method to calculate the unknown variables of an MMC.

3.2 | Parameters analysis of MMC phasor model

In the MMC-HVDC system of half-bridge SMs, the DC modulation index M_{dc} is generally constant at 1. In a full-bridge or a hybrid system, M_{dc} is approximately equal to 1 [23–26].

Equation (17) shows the parameter selection methods of the SM capacitor and the arm inductor [27]:

$$\begin{cases} C_0 = \frac{P_N}{3m_k N \omega_0 \epsilon_{\text{cap}} U_{\text{capN}}^2} \left[1 - \left(\frac{m_k \cos \varphi}{2} \right)^2 \right]^{3/2} \\ L_0 = \frac{1}{8\omega_0^2 C_0 U_{\text{capN}}} \left(\frac{P_N}{3I_{\text{km}}} + U_{\text{dcN}} \right) \end{cases}, \quad (17)$$

where U_{capN} is the rated capacitor voltage, ϵ_{cap} is the designed ripple percentage of capacitor voltages, I_{km} is the designed peak value of the circulating circulation, ω_0 is the fundamental angular frequency, and $\cos \varphi$ is the power factor, taken as 1.

If we define $\epsilon_{I_{\text{km}}} = I_{\text{km}}/(\sqrt{2}I_{\text{vN}})$, which is the designed percentage of the circulating circulation peak value, there is the function relation shown by Equation (18) between X_{L0} and X_{Ceq} :

$$\frac{X_{L0}}{X_{\text{Ceq}}} = \frac{1}{8} \left(1 + \frac{m_k}{4\epsilon_{I_{\text{km}}}} \right) > \frac{1}{8}. \quad (18)$$

Ignoring the second-order modulation components of the switching function, because its index M_2 is usually small, the equivalent reactance X_{MMC} can be simplified to Equation (19). It illustrates that $X_{\text{MMC}} < 0$ when the fundamental modulation index M_{e} varies between 0 and 1, which means the reactance in the MMC steady-state model is capacitive.

$$\begin{aligned} X_{\text{MMC}} &= -\frac{X_{\text{Ceq}}}{64} \left[8M_{\text{dc}}^2 - 3M_{\text{e}}^2 + \frac{6(3M_{\text{dc}}^2 - M_{\text{e}}^2)X_{\text{Ceq}}}{32X_{L0} - (2M_{\text{dc}}^2 + M_{\text{e}}^2)X_{\text{Ceq}}} M_{\text{e}}^2 \right] \\ &< -\frac{X_{\text{Ceq}}}{64} \left[8M_{\text{dc}}^2 - 3M_{\text{e}}^2 + \frac{6M_{\text{e}}^2(3M_{\text{dc}}^2 - M_{\text{e}}^2)}{4 - 2M_{\text{dc}}^2 + M_{\text{e}}^2} \right] < 0 \end{aligned} \quad (19)$$

3.3 | Model of an MMC connecting to an AC grid

The equivalent model of an MMC connecting to an AC grid is shown in Figure 2. The MMC is described by an equivalent capacitor X_{MMC} in series with the equivalent voltage source $\mathbf{E}_c$, as shown inside the blue box. The AC grid is equivalent to a Thevenin model of the equivalent voltage source $\mathbf{E}_s$ in series with the equivalent resistance $Z_s \angle \theta_s$. The value of Z_s equals the reciprocal of the AC grid's short-circuit ratio (SCR). X_T and k_T are the reactance and the tap of the converter transformer, respectively. $\mathbf{U}_t$ and $\mathbf{I}_t$ are the voltage phasor and

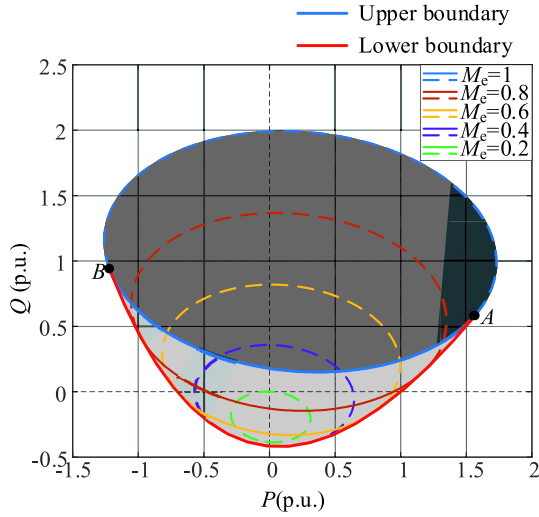


FIGURE 3 P - Q curves and operation boundaries

$$J = \begin{bmatrix} \frac{\partial P}{\partial U_{td}} & \frac{\partial P}{\partial U_{tq}} \\ \frac{\partial Q}{\partial U_{td}} & \frac{\partial Q}{\partial U_{tq}} \end{bmatrix}. \quad (26)$$

It is derived that the determinant of the Jacobian equals 0. The proof is shown in Appendix A. Thus, the envelope of the family of P - Q curves is equivalent to the curve corresponding to the condition that the Jacobian of the power flow equation is singular.

Plot P - Q curves with various M_e in its defined domain, as shown in Figure 3. The case shown in Figure 3 is of $\text{SCR} = 1.5$ and $\theta_s = 80^\circ$. In the P - Q chart, $P > 0$ means that the MMC operates in the inverter state, and $P < 0$ reflects that the MMC operates as a rectifier; $Q > 0$ illustrates that the reactive power output of MMC is inductive, while $Q < 0$ means MMC outputs capacitive reactive power.

The lower boundary, as shown by the solid red line, is determined by the singularity condition of the Jacobian of the power flow equation, on which the points follow as Equation (27).

$$E_s^T E_s = 2E_s^T V_t. \quad (27)$$

A P - Q curve is divided into two parts by its tangent points with the lower boundary if the tangent points exist, where the saddle-node bifurcation occurs. Considering the control process of MMC and the voltage stability at the PCC, the points in the solid lines are high voltage equilibrium and are stable; conversely, the points located in the dashed lines are unstable [28]. Some P - Q curves with small M_e have no tangent point with the lower boundary, where all the operating points are unstable. MMC can operate at the dashed line points by directly controlling the fundamental

modulation indices M_{ed} and M_{eq} , but cannot adjust the power outputs at their target value. Therefore, without considering the security constraints of the voltage offset and the current overload, the upper boundary of the power operation region of an MMC is determined by $M_e = M_{e\text{-max}}$, as shown by the solid blue line.

The upper boundary is related to the MMC's parameters, while the lower boundary is determined only by the AC grid's parameters. The region enclosed by the two boundaries, shown as the light grey area, is the feasible operation region of an MMC connecting to an AC grid constrained by the converter. The dark-filled area consists only of the dashed line part of the P - Q curves, where the MMC could operate by directly generating the controller outputs; that is, M_{dc} , M_{ed} , M_{eq} , M_{2d} , and M_{2q} , but this condition is abnormal. Points A and B are the tangent points of the upper and lower boundaries, where the MMC delivers and receives the maximum active power, respectively.

4.1.2 | Effect of the second-order modulation components on the operation region boundary

Substituting $M_e^2 = M_{ed}^2 + M_{eq}^2$ and $M_2^2 = M_{2d}^2 + M_{2q}^2$ into Equation (16), note that both the derivatives of X_{MMC} and $\|E_c\|$ with respect to θ_2 can factor out the item of $\sin(\theta_2 - 2\theta_c)$. The detailed expressions of X_{MMC} , $\|E_c\|$ and their derivatives are shown by Equations (B-9) and (B-10) in Appendix B. The derivatives of P and Q with respect to θ_2 can factor out the item of $\sin(\theta_2 - 2\theta_c)$ as well, according to Equation (28), which means that when $\theta_2 = 2\theta_c$ or $\theta_2 = 2\theta_c + \pi$, the upper boundary is the highest and the size of the operation region is the largest, if M_2 is specified.

$$\begin{cases} \frac{dP}{d\theta_2} = \frac{\partial P}{\partial X_{\text{MMC}}} \frac{dX_{\text{MMC}}}{d\theta_2} + \frac{\partial P}{\partial \|E_c\|} \frac{d\|E_c\|}{d\theta_2} \\ \frac{dQ}{d\theta_2} = \frac{\partial Q}{\partial X_{\text{MMC}}} \frac{dX_{\text{MMC}}}{d\theta_2} + \frac{\partial Q}{\partial \|E_c\|} \frac{d\|E_c\|}{d\theta_2} \end{cases} \quad (28)$$

Thus, substituting Equation (29) into Equation (22) and calculating it with θ_c changing from 0 to 2π , the upper boundary can be determined:

$$\begin{cases} M_{2d} = -M_2 \frac{M_{ed}^2 - M_{eq}^2}{M_e^2} \\ M_{2q} = -M_2 \frac{2M_{ed}M_{eq}}{M_e^2} \end{cases} \quad \text{or} \quad \begin{cases} M_{2d} = M_2 \frac{M_{ed}^2 - M_{eq}^2}{M_e^2} \\ M_{2q} = M_2 \frac{2M_{ed}M_{eq}}{M_e^2} \end{cases} \quad (29)$$

Figure 4 shows the boundaries of the power region with various second-order modulation indices M_2 when the SCR is

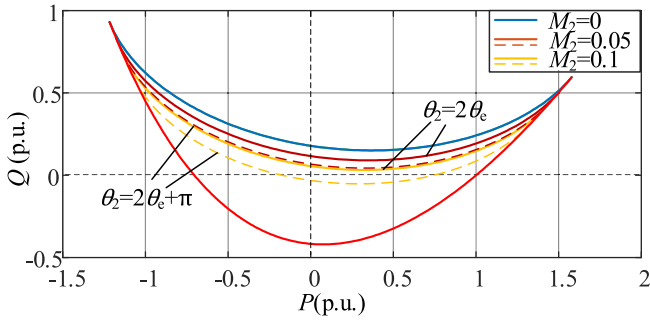


FIGURE 4 Operation boundaries with various M_2

1.5 and θ_s is 80° . The solid line represents the case of $\theta_2 = 2\theta_c$, and the dashed line corresponds to $\theta_2 = 2\theta_c + \pi$. Under this condition, the upper boundary is the highest when $\theta_2 = 2\theta_c$. However, the second-order modulation index M_2 occupies the range of the fundamental modulation index M_c , reducing the size of the operation region. Therefore, the operation region is maximised when $M_2 = 0$; that is, when there is no circulating current control.

4.1.3 | Effect of the converter parameters on the operation region boundary

It is noted from Figure 2 that the equivalent reactance X_{MMC} could affect the power outputs of MMC, which means both the arm inductor L_0 and the capacitance C_0 affect the operation region of MMC. According to Equation (19), reducing the submodule capacitance C_0 can reduce X_{MMC} . However, the arm inductor L_0 should be increased with the reduction of C_0 according to Equation (17) to guarantee the peak value of the circulating current. Therefore, we analyse the influence of the designed parameters ϵ_{cap} and ϵ_{lkm} on the maximum active power transmission. Figure 5 shows the results when the SCR is 1.5 and θ_s is 80° .

As shown in Figure 4, when ϵ_{lkm} is large (that is, the arm inductor L_0 is small), reducing the submodule capacitance C_0 can increase the power transmission of the MMC. When ϵ_{lkm} is small, reducing C_0 will reduce the MMC power transmission. When ϵ_{lkm} is very small and ϵ_{cap} is very large, the maximum power transmission will be significantly reduced, which may not satisfy power transmission requirements. Therefore, when designing the converter parameters, the requirements for inductance and capacitance parameters by the maximum power transmission should also be checked.

4.2 | Operation region considering the current constraint

To protect the semiconductors in an MMC from overload, an additional current limiter is included in the current controller

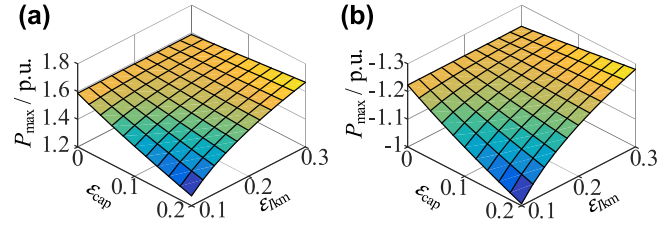


FIGURE 5 Influence of converter parameters on the maximum power transmission: (a) inverter, (b) rectifier

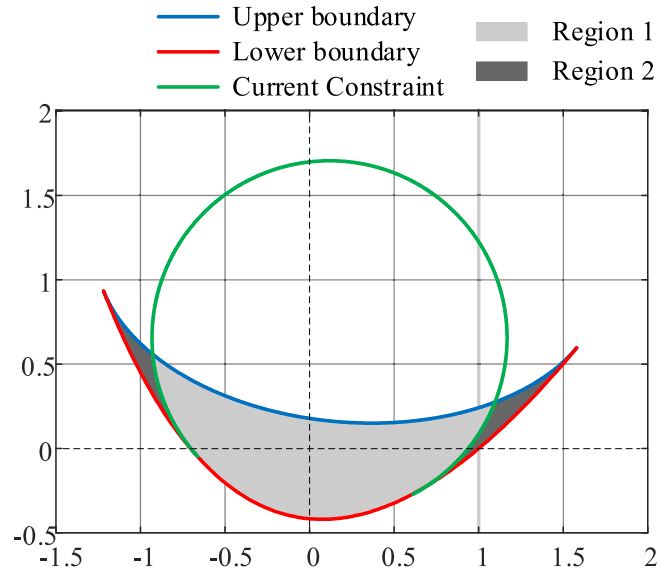


FIGURE 6 Operation region considering security constraints

to keep the current I_v within the rated current of the MMC [8]. Similar to the analysis of converter constraint, according to Equation (22), the equation of P - Q curves can also be written as $P = P(I_v, \varphi_v)$ and $Q = Q(I_v, \varphi_v)$, where I_v and φ_v are the magnitude and the angel of the current I_v , respectively. Therefore, the region satisfying the current constraint comprises a family of P - Q curves of various I_v within the range of $[0, I_{v_max}]$, where I_{v_max} is the current limit. The boundaries of the current constraint consist of the P - Q curve with $I_v = I_{v_max}$ and the curve determined by the condition that the power flow Jacobian is singular.

The operation region considering the security constraints is shown in Figure 6, where SCR is 1.5, θ_s is 80° , and I_{v_max} is 1.18. The red curve is the lower boundary of $|J| = 0$, the blue curve is the upper boundary of the converter constraint, and the green curve is the boundary of the current constraint. Region 1, shown in light grey, is where the operating states meet both the converter constraint and the current constraint. Region 2, located by both the left and right sides of region 1, is where only the current constraint is violated. MMC could operate in region 2 by using oversized insulated-gate bipolar

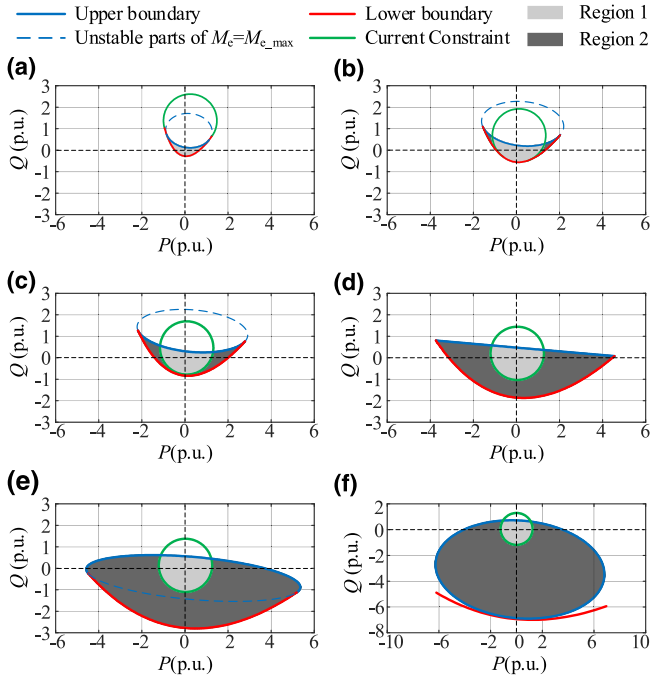


FIGURE 7 Influence of SCR on power operation region. (a) SCR is 1, (b) SCR is 2, (c) SCR is 3, (d) SCR is 6.68, (e) SCR is 10, (f) SCR is 25. SCR, short-circuit ratio

transistors to provide system support features during emergency conditions [29]. However, an MMC connecting to an AC grid cannot operate stably in the region outside the union of regions 1 and 2.

4.3 | Effect of the SCR on the operation region

The power operation regions under various SCR conditions are shown in Figure 7. The SCRs are set to 1, 2, 3, 6.68, 10, and 25, and θ_s is set as 80° . In this example, I_{v_max} is set as 1.18, M_{e_max} as 1, and M_2 as 0, and the $|Z_\Sigma|$ is approximately equal to the inverse of 6.68. It could be noted that both region 1 and region 2 expand with the increase in the SCR.

In Figure 7a,b, where the SCR is relatively small, the lower boundaries of region 1 are determined by $|J| = 0$, and both the P - Q curves of $M_e = M_{e_max}$ and of $I_v = I_{v_max}$ have tangent points with the lower boundary. If the SCR is quite small, as shown in Figure 7a, the dominating constraint is only the converter constraint, and the MMC could always transmit power without violating the current constraint. After the SCR increases, as shown in Figure 7c-f, the curve of $I_v = I_{v_max}$ will have no tangent point with the curve of $|J| = 0$; that is, region 1 can be determined by $M_e = M_{e_max}$ and $I_v = I_{v_max}$.

Figure 7a-e shows that the lower boundaries of region 2 are determined by $|J| = 0$, and the upper boundaries are the stable parts of the P - Q curves with $M_e = M_{e_max}$, where the

TABLE 1 Parameters of the simulation system

Parameter	Value
EMF of AC grid (kV)	551.25
SCR	1.5
Impedance angle of AC grid ($^\circ$)	80
Rated frequency of AC grid (Hz)	50
Transformer ratio (kV: kV: kV)	525:66:437.23
Transformer winding configuration	$Y_G/\Delta/Y$
Transformer power rating (MVA)	1380
Leakage reactance	24, 14, 8
DC voltage (kV)	± 420
Rated active power of MMC (MW)	1250
The number of SMs in each arm	500
Capacitance of SM (μF)	11,000
Arm inductor (H)	0.14

Abbreviations: AC, alternating current; DC, direct current; EMF, electromotive force; MMC, modular multilevel converter; MVA, Megavolt ampere; MW, Megawatt; SCR, short-circuit ratio; SM, submodule.

lower parts are stable when $SCR < 1/|Z_\Sigma|$ and the upper parts are stable when $SCR > 1/|Z_\Sigma|$. The unstable parts of the P - Q curves with $M_e = M_{e_max}$ are indicated by the dashed blue line. In particular, when the SCR is quite large, as shown in Figure 7f, the P - Q curve of $M_e = M_{e_max}$ has no tangent points with the curve of $|J| = 0$; that is, the singularity of Jacobian is no longer the boundary condition of the MMC steady-state power region, and all points inside regions 1 and 2 are stable.

5 | SIMULATION VALIDATION

To verify the proposed model and power region calculation method, we established a single-terminal half-bridge MMC-HVDC simulation system in PSCAD/EMTDC; the system parameters are shown in Table 1.

5.1 | Verification of the steady-state phasor model

We calculated the proposed model in this paper, the models in [9, 10, 13], and the accurate models in [8, 17] using Matlab2016, and compared their results with the simulation results in PSCAD/EMTDC. In this example, the DC modulation index is set as 1, the fundamental modulation index M_e as 0.95, and the second-order modulation index M_2 as 0. Changing θ_e from 0 to 2π by the step $\Delta\theta_e$ as $\pi/1800$, the CPU time of the three models in Matlab2016 is 0.1716, 0.1035, and 110.7943 s, respectively. The comparison between the calculation results

and the simulation results in PSCAD/EMTDC is shown in Figure 8.

There are 12 sets of comparison curves, including the active power P and reactive power Q , the d - and q -components of voltage U , the d - and q -components of the common voltage U_v , the d - and q -components of the current I_v , the DC component of the equivalent arm capacitor voltage $U_{\Sigma \text{ cap0}}$, the DC component of arm currents I_{diff0} , and the d - and q -components of the second-order components of current I_{diff2} . Only the former eight curves can be obtained by the model [9, 10, 13]. The solid blue lines denote the calculated results using the proposed method; the black balls denote the simulated results. The average relative errors between the calculation results and the simulation results are 3.2536%, 2.7387%, 2.8025%, 2.7007%, 3.3567%, 3.5763%, 1.1105%, 1.7416%, 0.1079%, 1.6837%, 3.5879%, and 4.7986%, respectively, which are all less than 5%. In other words, the proposed model has high accuracy. The calculated results of the models [9, 10, 13] are denoted by the dashed red lines, which do not fit well with the simulation results because the models cannot describe the coupling characteristic inside the converter. The results of models [8, 17] are denoted by the solid red lines, which is accurate.

Thus, it is verified that the proposed model in this paper has high accuracy and has apparent advantages in computational efficiency due to its open-loop calculation process.

5.2 | Verification of determination method of the operation region

To verify the determination method of the upper boundary, we set M_{e_max} as 0.95 and calculate three examples of $\theta_2 = 2\theta_e$, $\theta_2 = 2\theta_e + \pi$, and $\theta_2 = 2\theta_e + \pi/2$. The calculation results denoted by the solid lines and the simulation results denoted by

the black balls are shown in Figure 9. The average relative errors are 1.0827%, 1.1573%, and 1.0318%, respectively. The upper boundary of $\theta_2 = 2\theta_e$ is at the highest position, which matches the analysis results in Section 4.1.2. Thus, it is verified that the proposed determination method for the upper boundary of the power operation region is correct and can be used to analyse the influence of the second-order modulation components on the operation region.

To verify the calculation method of the lower boundary and the current constraint boundary, two typical operating points were simulated in PSCAD/EMTDC, where the first point was in region 2 and the second was outside the sum of regions 1 and 2. Calculating by Equation (27), the active power was 1.006 when setting the reactive power to 0, which is on the lower boundary. Figure 10 shows the response to the change in active power from 1 to 1.01 at $t = 20$ s. We found that the system is stable at the first point although the current I_t exceeds its rated value (1.375 kA), but the system would be unstable at the second point, which confirms the theoretical calculation of the power operation region.

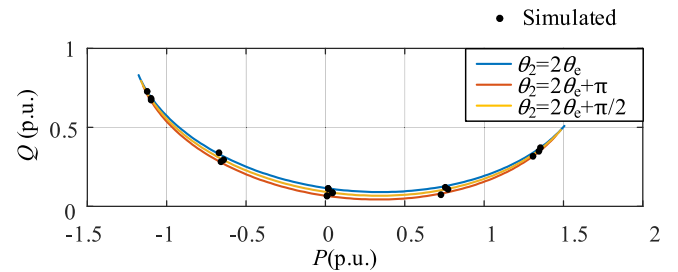


FIGURE 9 Verification of the calculation method for the upper boundary of the power operation region

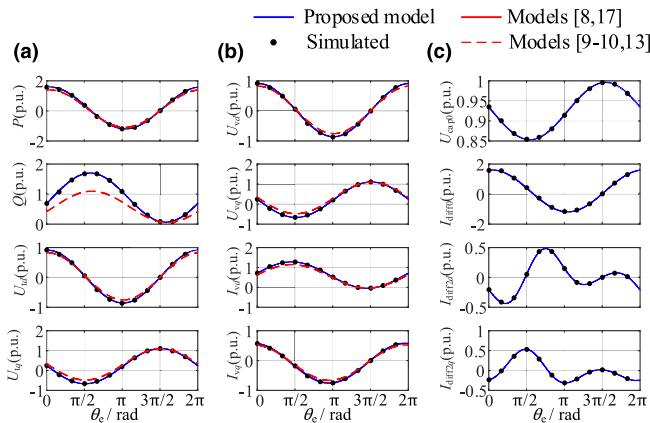


FIGURE 8 Verification and comparison for the proposed MMC model. (a) Curves of the variables at the grid side, (b) curves of the variables at the valve side, (c) curves of the variables inside the MMC. MMC, modular multilevel converter

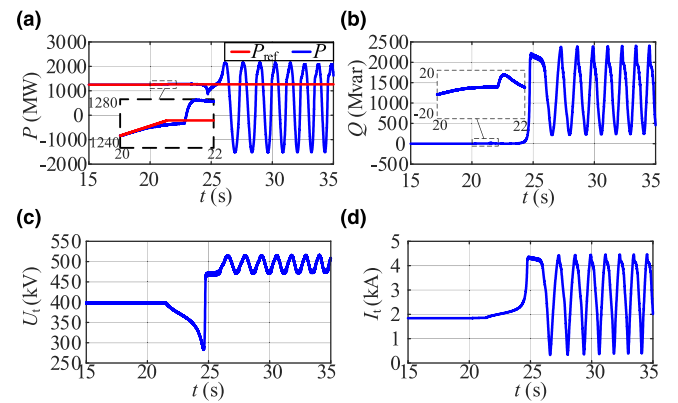


FIGURE 10 Comparative verification curves. (a) active power, (b) reactive power, (c) voltage root mean square (RMS) of grid side, (d) current RMS of grid side

6 | CONCLUSIONS

Considering the different operating characteristics of the MMC and the two-level VSC, we propose a novel steady-state phasor model of an MMC done by deriving the function relation between the voltage and current outputs in the d - q frame. We also propose an open-loop calculation method for the steady-state power operation region of MMC based on the model and study its influencing factors.

The main conclusions are as follows:

1. The MMC is equivalent to an equivalent capacitor in series with an equivalent voltage source. In contrast, there is no equivalent capacitor at the AC-side equivalent model of a two-level VSC. The equivalent capacitor in the MMC model reflects the influence of converter internal coupling characteristics on the AC outputs; that is, the difference between an MMC and a two-level VSC.
2. The limit power operation region is constrained by the converter, which is the most fundamental constraint. The upper boundary is determined by $M_e = M_{e_max}$ and the lower boundary is determined by the condition that the Jacobian of the power flow equation is singular. The lower boundary is determined only by the AC grid parameters, but the upper boundary can be affected by the MMC parameters and the circulating current control mode.
3. Considering the current constraint of MMC, the operation region of MMC is divided into two parts: region 1 is where the operating states meet both the converter constraint and the current constraint, and region 2 is where only the current constraint is violated. The SCR has an essential influence on the size and the dominating constraints of the MMC operation region.

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APPENDIX A

This section shows the derivation that the determinant of the Jacobian of the power flow equation equals 0 from the condition of the envelope.

The condition of the envelope shown by Equation (25) can be rewritten as

$$\begin{cases} \frac{\partial P}{\partial M_e} = k \frac{\partial P}{\partial \theta_e} \\ \frac{\partial Q}{\partial M_e} = k \frac{\partial Q}{\partial \theta_e} \end{cases}, \quad k \neq 0 \quad (\text{A-1})$$

Equation (A-2) gives the function relation between the partial derivatives in Equation (A-1) and the elements in the Jacobian of power flow equation.

$$\begin{cases} \frac{\partial P}{\partial M_e} = \frac{\partial P}{\partial U_{td}} \frac{\partial U_{td}}{\partial M_e} + \frac{\partial P}{\partial U_{tq}} \frac{\partial U_{tq}}{\partial M_e} \\ \frac{\partial P}{\partial \theta_e} = \frac{\partial P}{\partial U_{td}} \frac{\partial U_{td}}{\partial \theta_e} + \frac{\partial P}{\partial U_{tq}} \frac{\partial U_{tq}}{\partial \theta_e} \\ \frac{\partial Q}{\partial M_e} = \frac{\partial Q}{\partial U_{td}} \frac{\partial U_{td}}{\partial M_e} + \frac{\partial Q}{\partial U_{tq}} \frac{\partial U_{tq}}{\partial M_e} \\ \frac{\partial Q}{\partial \theta_e} = \frac{\partial Q}{\partial U_{td}} \frac{\partial U_{td}}{\partial \theta_e} + \frac{\partial Q}{\partial U_{tq}} \frac{\partial U_{tq}}{\partial \theta_e} \end{cases} \quad (\text{A-2})$$

Substituting Equation (A-1) into Equation (A-2), the partial derivatives in Jacobian of power flow equation can be expressed by Equation (A-3).

$$\begin{cases} \frac{\partial P}{\partial U_{td}} \left(\frac{\partial U_{td}}{\partial M_e} - k \frac{\partial U_{td}}{\partial \theta_e} \right) = - \frac{\partial P}{\partial U_{tq}} \left(\frac{\partial U_{tq}}{\partial M_e} - k \frac{\partial U_{tq}}{\partial \theta_e} \right) \\ \frac{\partial Q}{\partial U_{td}} \left(\frac{\partial U_{td}}{\partial M_e} - k \frac{\partial U_{td}}{\partial \theta_e} \right) = - \frac{\partial Q}{\partial U_{tq}} \left(\frac{\partial U_{tq}}{\partial M_e} - k \frac{\partial U_{tq}}{\partial \theta_e} \right) \end{cases} \quad (\text{A-3})$$

Thus, Equation (A-4) can be obtained.

$$\frac{\partial P}{\partial U_{td}} \frac{\partial Q}{\partial U_{tq}} = \frac{\partial P}{\partial U_{tq}} \frac{\partial Q}{\partial U_{td}} \quad (\text{A-4})$$

The determinant of Jacobian of the power flow equation is calculated by Equation (A-5), which equals 0.

$$|J| = \begin{vmatrix} \frac{\partial P}{\partial U_{ud}} & \frac{\partial P}{\partial U_{uq}} \\ \frac{\partial Q}{\partial U_{ud}} & \frac{\partial Q}{\partial U_{uq}} \end{vmatrix} = \frac{\partial P}{\partial U_{ud}} \frac{\partial Q}{\partial U_{uq}} - \frac{\partial P}{\partial U_{uq}} \frac{\partial Q}{\partial U_{ud}} = 0 \quad (\text{A-5})$$

APPENDIX B

The detailed expression of matrices in Equation (13) are shown by Equations (B-1)–(B-5).

$$\mathbf{A} = \begin{bmatrix} -\frac{M_{ed}}{8} & -\frac{M_{eq}}{8} \\ \frac{-M_{ed}M_{2q} + M_{eq}(4M_{dc} + M_{2d})}{32\omega C_{eq}} & \frac{M_{ed}(4M_{dc} - M_{2d}) - M_{eq}M_{2q}}{32\omega C_{eq}} \\ -\frac{M_{ed}M_{2q} + M_{eq}(3M_{dc} + M_{2d})}{16\omega C_{eq}} & -\frac{M_{ed}(3M_{dc} - M_{2d}) + M_{eq}M_{2q}}{16\omega C_{eq}} \\ \frac{M_{ed}(3M_{dc} + M_{2d}) - M_{eq}M_{2q}}{16\omega C_{eq}} & \frac{M_{ed}M_{2q} - M_{eq}(3M_{dc} - M_{2d})}{16\omega C_{eq}} \end{bmatrix} \quad (\text{B-1})$$

$$\mathbf{B} = \begin{bmatrix} 0 & \frac{M_{dc}}{2} & \frac{M_{2d}}{4} & \frac{M_{2q}}{4} \\ \frac{M_{dc}}{2} & 0 & \frac{M_{dc}M_{2q} + 2M_{ed}M_{eq}}{16\omega C_{eq}} & \frac{M_{dc}M_{2d} + M_{ed}^2 - M_{eq}^2}{16\omega C_{eq}} \\ \frac{M_{2d}}{2} & \frac{M_{dc}M_{2q} + 2M_{ed}M_{eq}}{8\omega C_{eq}} & 0 & \frac{2M_{dc}^2 + M_{ed}^2 + M_{eq}^2}{16\omega C_{eq}} - 2\omega L_0 \\ \frac{M_{2q}}{2} & \frac{M_{dc}M_{2d} + M_{ed}^2 - M_{eq}^2}{8\omega C_{eq}} & 2\omega L_0 - \frac{2M_{dc}^2 + M_{ed}^2 + M_{eq}^2}{16\omega C_{eq}} & 0 \end{bmatrix} \quad (\text{B-2})$$

$$\mathbf{C} = [0 \quad -1/2 \quad 0 \quad 0]^T \quad (\text{B-3})$$

$$\mathbf{D} = \begin{bmatrix} 0 & \frac{8M_{dc}^2 + (M_{ed}^2 + M_{eq}^2) - 2(M_{2d}^2 + M_{2q}^2)}{32\omega C_{eq}} \\ \frac{8M_{dc}^2 + (M_{ed}^2 + M_{eq}^2) - 2(M_{2d}^2 + M_{2q}^2)}{32\omega C_{eq}} & 0 \end{bmatrix} \quad (\text{B-4})$$

$$\mathbf{E} = \begin{bmatrix} \frac{M_{ed}}{2} & \frac{M_{ed}M_{2q} - M_{eq}(4M_{dc} + M_{2d})}{16\omega C_{eq}} & \frac{M_{ed}(3M_{dc} + M_{2q}) + M_{eq}M_{2d}}{16\omega C_{eq}} & \frac{M_{ed}(3M_{dc} + M_{2d}) - M_{eq}M_{2q}}{16\omega C_{eq}} \\ \frac{M_{eq}}{2} & \frac{M_{ed}M_{2d} - M_{eq}(4M_{dc} - M_{2q})}{16\omega C_{eq}} & \frac{M_{ed}(3M_{dc} - M_{2d}) + M_{eq}M_{2q}}{16\omega C_{eq}} & \frac{M_{ed}(3M_{dc} - M_{2q}) - M_{eq}M_{2d}}{16\omega C_{eq}} \end{bmatrix} \quad (\text{B-5})$$

$$\mathbf{F} = [0 \quad 0 \quad 0 \quad 0]^T \quad (\text{B-6})$$

The detailed expression of matrices in Equation (21) are shown by Equations (B-7) and (B-8).

$$\mathbf{S}_B = \begin{bmatrix} \frac{[\lambda_1 M_{eq} + \lambda_2 (M_{ed}M_{2q} - M_{eq}M_{2d})] m_k X_{Ceq}}{64(\zeta_1 + \zeta_2 + \zeta_3)} & \frac{[-\lambda_1 M_{ed} + \lambda_2 (M_{ed}M_{2d} + M_{eq}M_{2q})] m_k X_{Ceq}}{64(\zeta_1 + \zeta_2 + \zeta_3)} \\ \frac{[\lambda_3 M_{ed} + 3M_{dc}(M_{ed}M_{2d} + M_{eq}M_{2q})] X_{Ceq}}{m_k(\zeta_1 + \zeta_2 + \zeta_3)} & \frac{[\lambda_3 M_{eq} + 3M_{dc}(M_{ed}M_{2q} - M_{eq}M_{2d})] X_{Ceq}}{m_k(\zeta_1 + \zeta_2 + \zeta_3)} \\ \frac{[2M_{ed}(3M_{dc}^2 - M_{ed}^2 + M_{eq}^2) + M_{2q}(6M_{dc}M_{eq} + M_{ed}M_{2q} - M_{eq}M_{2d})] X_{Ceq}}{4(\zeta_1 + \zeta_2 + \zeta_3)} & \frac{[2M_{eq}(3M_{dc}^2 + M_{ed}^2 - M_{eq}^2) + M_{2q}(6M_{dc}M_{ed} + M_{ed}M_{2d} + M_{eq}M_{2q})] X_{Ceq}}{4(\zeta_1 + \zeta_2 + \zeta_3)} \\ \frac{[2M_{eq}(3M_{dc}^2 - 2M_{ed}^2) + M_{2d}(6M_{dc}M_{eq} + M_{ed}M_{2q} - M_{eq}M_{2d})] X_{Ceq}}{4(\zeta_1 + \zeta_2 + \zeta_3)} & \frac{[2M_{ed}(3M_{dc}^2 - 2M_{eq}^2) + M_{2d}(6M_{dc}M_{ed} + M_{ed}M_{2d} + M_{eq}M_{2q})] X_{Ceq}}{4(\zeta_1 + \zeta_2 + \zeta_3)} \end{bmatrix} \quad (\text{B-7})$$

$$\text{where } \begin{cases} \lambda_1 = M_{dc}[8M_{dc}^2 - 2(M_{ed}^2 + M_{eq}^2) + 2(M_{2d}^2 + M_{2q}^2)]X_{Ceq} - 128M_{dc}X_{L0} \\ \lambda_2 = [8M_{dc}^2 - 2(M_{ed}^2 + M_{eq}^2)]X_{Ceq} - 32X_{L0} \\ \lambda_3 = [2M_{dc}^2 + (M_{ed}^2 + M_{eq}^2) - (M_{2d}^2 + M_{2q}^2)]X_{Ceq} - 32X_{L0} \end{cases}.$$

$$\mathbf{S}_C = \begin{bmatrix} \frac{[2M_{dc}^2 + 2(M_{ed}^2 + M_{eq}^2)]X_{Ceq} - 32X_{L0}}{\zeta_1 + \zeta_2 + \zeta_3} & 0 & \frac{16M_{2q}}{m_k(\zeta_1 + \zeta_2 + \zeta_3)} & -\frac{16M_{2d}}{m_k(\zeta_1 + \zeta_2 + \zeta_3)} \end{bmatrix}^T \quad (\text{B-8})$$

The expression of X_{MMC} and $\|\mathbf{E}_c\|$ is shown by Equation (B-9), where the derivative with respect to θ_2 is shown by Equation (B-10).

$$\begin{cases} X_{MMC} = -\frac{(\zeta_1 + \zeta_2 + \zeta_3)X_{Ceq}}{64(\zeta_1 + \zeta_2 + \zeta_3)} \\ \|\mathbf{E}_c\| = \frac{M_e U_{dc} \sqrt{(\zeta_1 + \zeta_2)^2 + (3X_{Ceq}M_{dc}^2M_2)^2 + 2(\zeta_1 + \zeta_2)(3X_{Ceq}M_{dc}^2M_2)\cos(\theta_2 - 2\theta_e)}}{m_k M_{dc}(\zeta_1 + \zeta_2 + \zeta_3)} \end{cases} \quad (\text{B-9})$$

$$\begin{cases} \frac{dX_{MMC}}{d\theta_2} = -\frac{M_e^2 M_2 X_{Ceq} \{X_{Ceq}(\zeta_1 + \zeta_2) - [32X_{L0} - (12M_{dc}^2 - 2M_e^2 + M_2^2)X_{Ceq}](\zeta_1 + \zeta_2)\}}{64(\zeta_1 + \zeta_2 + \zeta_3)^2} \sin(\theta_2 - 2\theta_e) \\ \frac{d\|\mathbf{E}_c\|}{d\theta_2} = -\frac{M_e M_2 U_{dc} X_{Ceq} [(3M_{dc}^2 - M_e^2)(\zeta_1 + \zeta_2)^2 - (3M_{dc}^2 M_e M_2 X_{Ceq})^2 - 3M_{dc}^2 M_e^2 M_2 X_{Ceq}(\zeta_1 + \zeta_2)\cos(\theta_2 - 2\theta_e)]}{m_k M_{dc}(\zeta_1 + \zeta_2 + \zeta_3)^2 \sqrt{(\zeta_1 + \zeta_2)^2 + (3X_{Ceq}M_{dc}^2M_2)^2 + 2(\zeta_1 + \zeta_2)(3X_{Ceq}M_{dc}^2M_2)\cos(\theta_2 - 2\theta_e)}} \sin(\theta_2 - 2\theta_e) \end{cases} \quad (\text{B-10})$$

$$\text{where } \left\{ \begin{array}{l} \varsigma_1 = 32M_{\text{dc}}X_{L0}(8M_{\text{dc}}^2 - 3M_{\text{e}}^2 - 2M_2^2) \\ \varsigma_2 = -M_{\text{dc}}X_{\text{Ceq}}(16M_{\text{dc}}^4 + 3M_{\text{e}}^4 + 2M_2^4 - 16M_{\text{dc}}^2M_{\text{e}}^2 - 12M_{\text{dc}}^2M_2^2 - 2M_{\text{e}}^2M_2^2) \\ \varsigma_3 = M_{\text{e}}^2M_2 \cos(\theta_2 - 2\theta_{\text{e}})[32X_{L0} - (12M_{\text{dc}}^2 - 2M_{\text{e}}^2 + M_2^2)X_{\text{Ceq}}] \\ \zeta_1 = 32M_{\text{dc}}X_{L0} \\ \zeta_2 = -M_{\text{dc}}(2M_{\text{dc}}^2 + M_{\text{e}}^2 - M_2^2)X_{\text{Ceq}} \\ \zeta_3 = M_{\text{e}}^2M_2X_{\text{Ceq}} \cos(\theta_2 - 2\theta_{\text{e}}) \end{array} \right. .$$